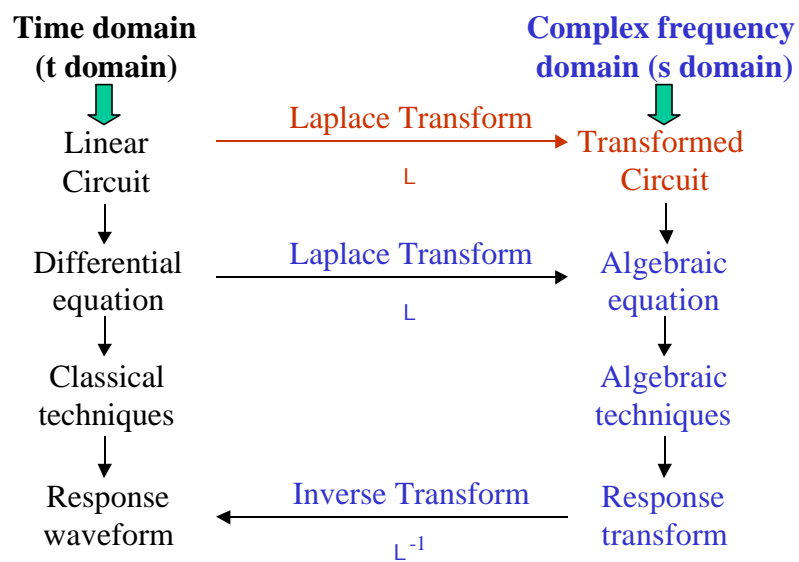
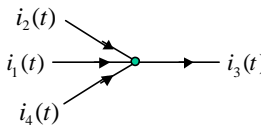
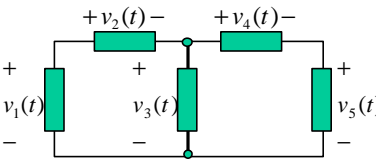


S-Domain Analysis

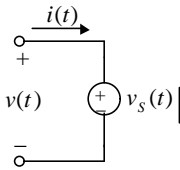

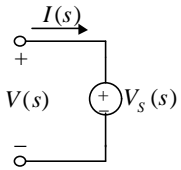
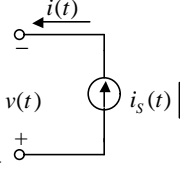

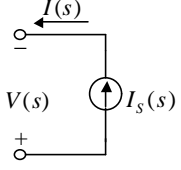
s-Domain Circuit Analysis

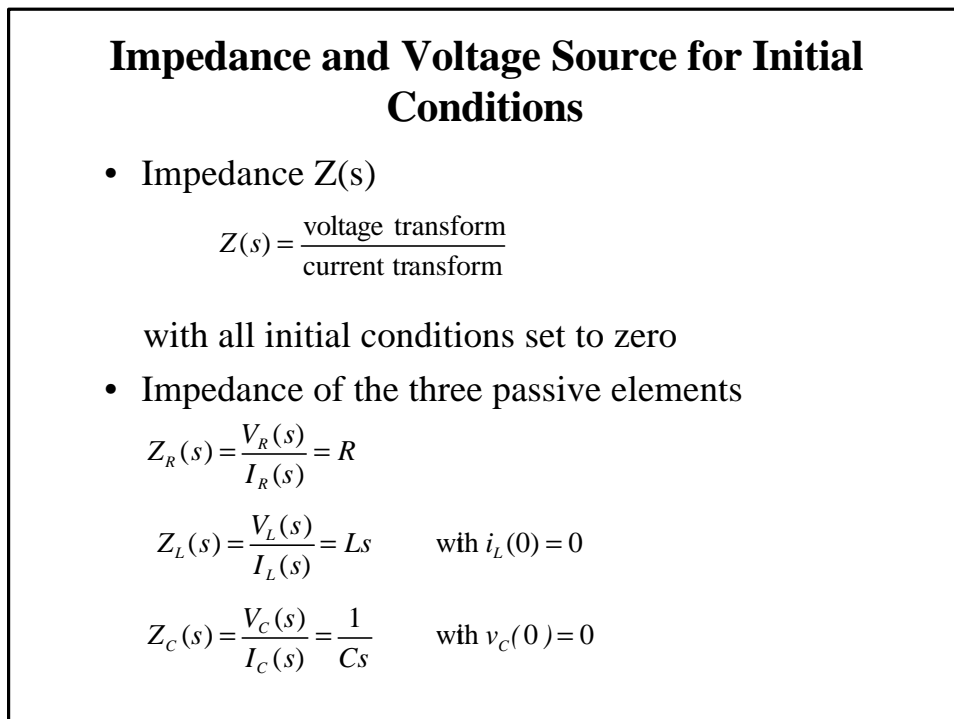
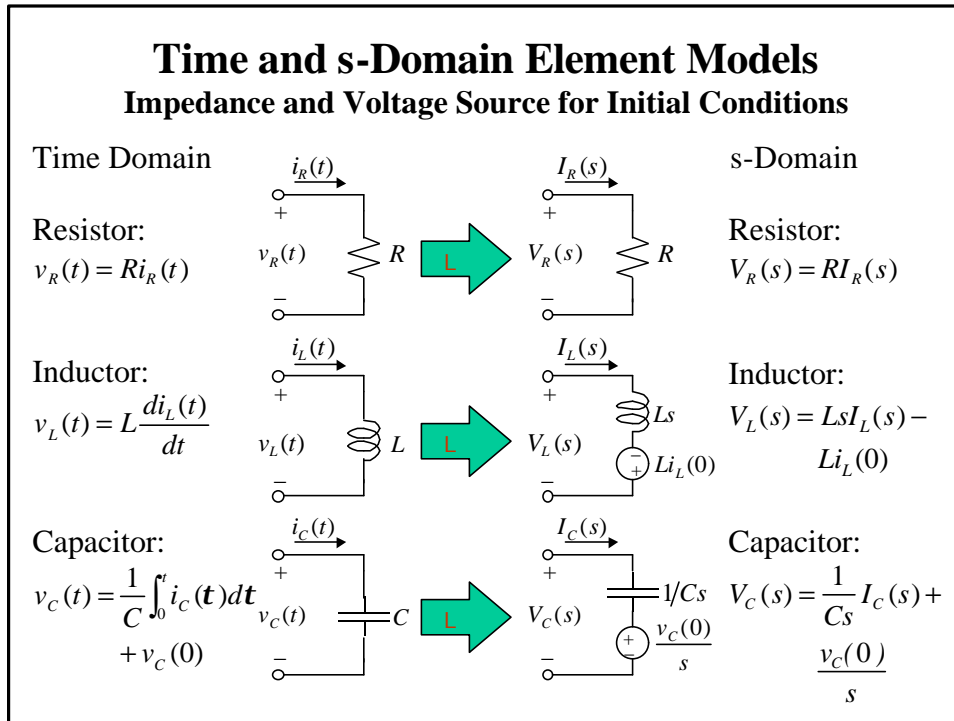


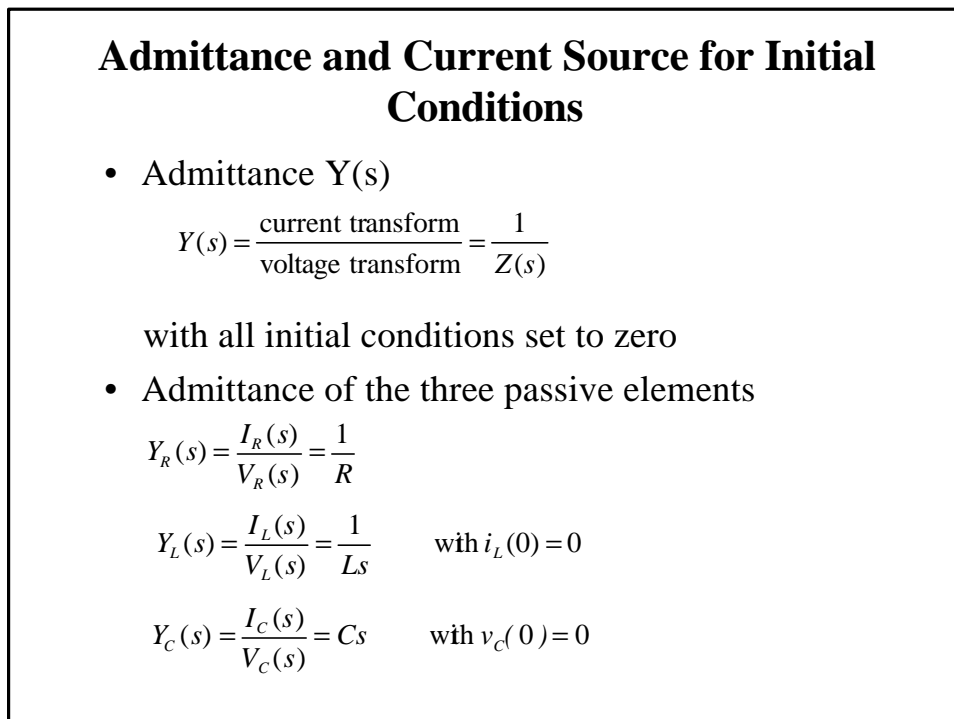
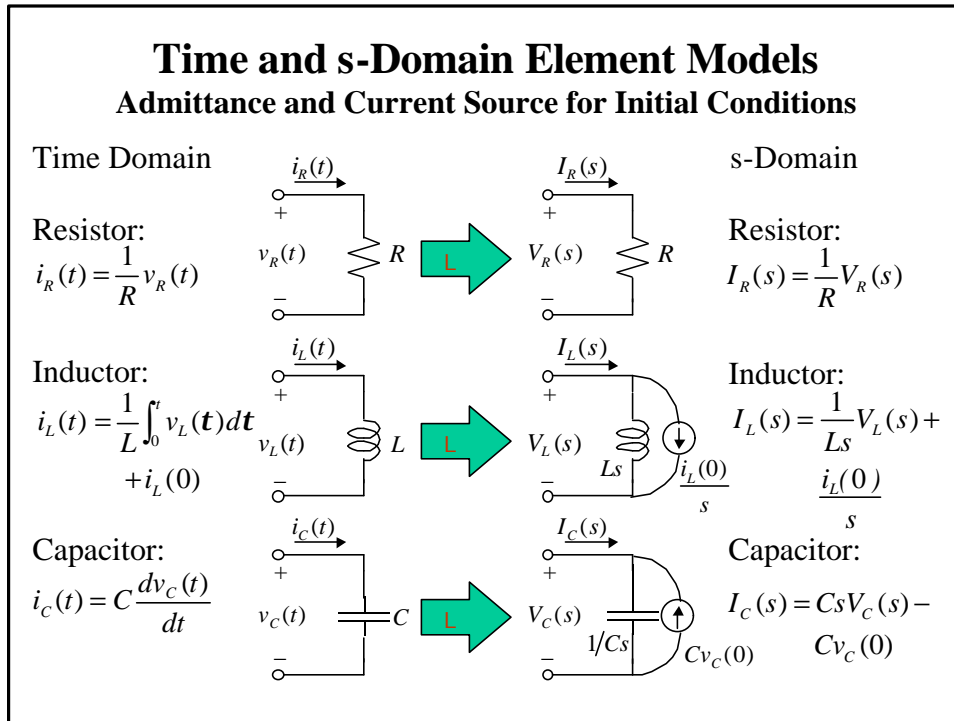
Kirchhoff's Laws in s-Domain

<p>t domain</p> <p>Kirchhoff's current law (KCL)</p> $i_1(t) + i_2(t) - i_3(t) + i_4(t) = 0$ <p>Kirchhoff's voltage law (KVL)</p> $-v_1(t) + v_2(t) + v_3(t) = 0$	<p>s domain</p>  $I_1(s) + I_2(s) - I_3(s) + I_4(s) = 0$  $-V_1(s) + V_2(s) + V_3(s) = 0$
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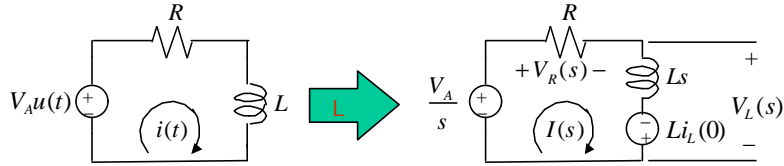
Signal Sources in s Domain

<p>t domain</p> <p>Voltage Source:</p> $v(t) = v_S(t)$ $i(t) = \text{depends on circuit}$ 		<p>s domain</p> <p>Voltage Source:</p> $V(s) = V_S(s)$ $I(s) = \text{depends on circuit}$ 
<p>Current Source:</p> $i(t) = i_S(t)$ $v(t) = \text{depends on circuit}$ 		<p>Current Source:</p> $I(s) = I_S(s)$ $V(s) = \text{depends on circuit}$ 





Example: Solve for Current Waveform $i(t)$



By KVL: $-\frac{V_A}{s} + V_R(s) + V_L(s) = 0$

Resistor: $V_R(s) = RI(s)$ Inductor: $V_L(s) = LsI(s) - Li_L(0)$

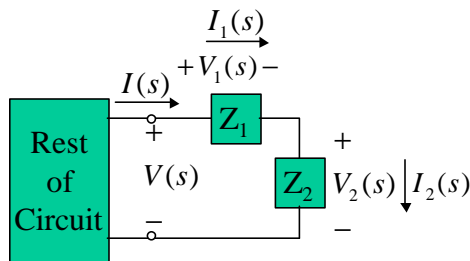
$$-\frac{V_A}{s} + RI(s) + LsI(s) - Li_L(0) = 0$$

$$I(s) = \frac{V_A/L}{s(s + R/L)} + \frac{i_L(0)}{s + R/L}$$

$$= \frac{V_A/R}{s} - \frac{V_A/R}{s + R/L} + \frac{i_L(0)}{s + R/L}$$

Inverse Transform: $i(t) = \left[\underbrace{\frac{V_A}{R}}_{\text{forced response}} - \underbrace{\frac{V_A}{R} e^{-\frac{R}{L}t}}_{\text{natural response}} + i_L(0)e^{-\frac{R}{L}t} \right] u(t)$

Series Equivalence and Voltage Division

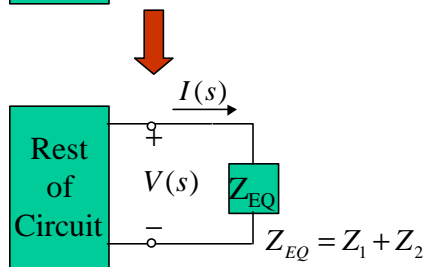


$$V_1(s) = Z_1(s)I_1(s) = Z_1(s)I(s)$$

$$V_2(s) = Z_2(s)I_2(s) = Z_2(s)I(s)$$

KVL: $V(s) = V_1(s) + V_2(s)$
 $= (Z_1(s) + Z_2(s))I(s)$

$\Rightarrow Z_{EQ}(s) = Z_1(s) + Z_2(s)$



$$V_1(s) = \frac{Z_1(s)}{Z_{EQ}(s)} V(s)$$

$$V_2(s) = \frac{Z_2(s)}{Z_{EQ}(s)} V(s)$$

Parallel Equivalence and Current Division

$I_1(s) = Y_1(s)V(s)$
 $I_2(s) = Y_2(s)V(s)$

KCL: $I(s) = I_1(s) + I_2(s)$
 $= (Y_1(s) + Y_2(s))V(s)$

$\Rightarrow Y_{EQ}(s) = Y_1(s) + Y_2(s)$

$I_1(s) = \frac{Y_1(s)}{Y_{EQ}(s)} I(s)$
 $I_2(s) = \frac{Y_2(s)}{Y_{EQ}(s)} I(s)$

$Y_{EQ} = Y_1 + Y_2$

Example: Equivalence Impedance and Admittance

Inductor current = 0 }
 capacitor voltage = 0 } at $t = 0$

Find equivalent impedance at A and B
 Solve for $v_2(t)$

$Y_{EQ1}(s) = \frac{1}{Z_{EQ1}(s)} = \frac{1}{R} + Cs = \frac{RCs + 1}{R}$

$Z_{EQ}(s) = Ls + Z_{EQ1}(s) = Ls + \frac{R}{RCs + 1}$

$= \frac{RLCs^2 + Ls + R}{RCs + 1}$

$V_2(s) = \frac{Z_{EQ1}(s)}{Z_{EQ}(s)} V_1(s)$

$= \frac{R}{RCLs^2 + Ls + R} V_1(s)$

Z_{EQ}

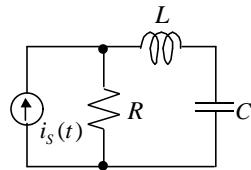
General Techniques for s-Domain Circuit Analysis

- Node Voltage Analysis (in s-domain)
 - Use Kirchhoff's Current Law (KCL)
 - Get equations of node voltages
 - Use current sources for initial conditions
 - Voltage source \longrightarrow current source
- Mesh Current Analysis (in s-domain)
 - Use Kirchhoff's Voltage Law (KVL)
 - Get equations of currents in the mesh
 - Use voltage sources for initial conditions
 - Current source \longrightarrow voltage source
(Works only for "Planar" circuits)

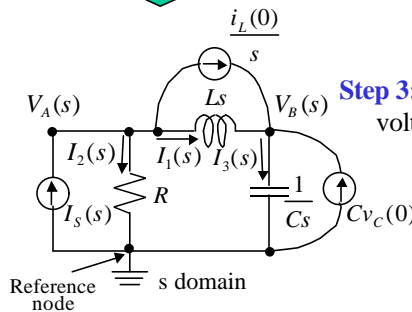
Formulating Node-Voltage Equations

- Step 0:** Transform the circuit into the s domain using current sources to represent capacitor and inductor initial conditions
- Step 1:** Select a reference node. Identify a node voltage at each of the non-reference nodes and a current with every element in the circuit
- Step 2:** Write KCL connection constraints in terms of the element currents at the non-reference nodes
- Step 3:** Use the element admittances and the fundamental property of node voltages to express the element currents in terms of the node voltages
- Step 4:** Substitute the device constraints from Step 3 into the KCL connection constraints from Step 2 and arrange the resulting equations in a standard form

Example: Formulating Node-Voltage Equations



t domain

Reference node
s domain

Step 0: Transform the circuit into the s domain using current sources to represent capacitor and inductor initial conditions

Step 1: Identify $N-1=2$ node voltages and a current with each element

Step 2: Apply KCL at nodes A and B:

$$\text{Node A: } I_S(s) - \frac{i_L(0)}{s} - I_1(s) - I_2(s) = 0$$

$$\text{Node B: } C v_C(0) + \frac{i_L(0)}{s} + I_1(s) - I_3(s) = 0$$

Step 3: Express element equations in terms of node voltages

$$I_1(s) = Y_L(s)[V_A(s) - V_B(s)] = \frac{1}{Ls}[V_A(s) - V_B(s)]$$

$$I_2(s) = Y_R(s)V_A(s) = G V_A(s) \text{ where } G = 1/R$$

$$I_3(s) = Y_C(s)V_B(s) = Cs V_B(s)$$

Formulating Node-Voltage Equations (Cont'd)

Step 2: Apply KCL at nodes A and B:

$$\text{Node A: } I_S(s) - \frac{i_L(0)}{s} - I_1(s) - I_2(s) = 0$$

$$\text{Node B: } C v_C(0) + \frac{i_L(0)}{s} + I_1(s) - I_3(s) = 0$$

Step 3: Express element equations in terms of node voltages

$$I_1(s) = Y_L(s)[V_A(s) - V_B(s)] = \frac{1}{Ls}[V_A(s) - V_B(s)]$$

$$I_2(s) = Y_R(s)V_A(s) = G V_A(s) \text{ where } G = 1/R$$

$$I_3(s) = Y_C(s)V_B(s) = Cs V_B(s)$$

Step 4: Substitute eqns. in Step 3 into eqns. in Step 2 and collect common terms to yield node-voltage eqns.

$$\text{Node A: } \left(G + \frac{1}{Ls}\right)V_A(s) - \left(\frac{1}{Ls}\right)V_B(s) = I_S(s) - \frac{i_L(0)}{s}$$

$$\text{Node B: } -\left(\frac{1}{Ls}\right)V_A(s) + \left(\frac{1}{Ls} + Cs\right)V_B(s) = C v_C(0) + \frac{i_L(0)}{s}$$

Solving s-Domain Circuit Equations

- Circuit Determinant: $\Delta(s) = \begin{vmatrix} G+1/Ls & -1/Ls \\ -1/Ls & Cs+1/Ls \end{vmatrix}$
 $= (G+1/Ls)(Cs+1/Ls) - (1/Ls)^2$
 $= \frac{GLCs^2 + Cs + G}{Ls}$

Depends on circuit element parameters: L, C, G=1/R,
not on driving force and initial conditions

- Solve for node A using Cramer's rule:

$$V_A(s) = \frac{\Delta_A(s)}{\Delta(s)} = \frac{\begin{vmatrix} I_s(s) + i_L(0)/s & -1/Ls \\ i_L(0)/s + Cv_C(0) & Cs + 1/Ls \end{vmatrix}}{\Delta(s)}$$

$$= \underbrace{\frac{(LCs^2 + 1)I_s(s)}{GLCs^2 + Cs + G}}_{\text{Zero State}} + \underbrace{\frac{-LCsi_L(0) + Cv_C(0)}{GLCs^2 + Cs + G}}_{\text{Zero input}}$$

Zero State Zero input
when initial condition when input sources
sources are turned off are turned off

Solving s-Domain Circuit Eqns. (Cont'd)

- Solve for node B using Cramer's rule:

$$V_B(s) = \frac{\Delta_B(s)}{\Delta(s)} = \frac{\begin{vmatrix} G+1/Ls & I_s(s) - i_L(0)/s \\ -1/Ls & i_L(0)/s + Cv_C(0) \end{vmatrix}}{\Delta(s)}$$

$$= \underbrace{\frac{I_s(s)}{GLCs^2 + Cs + G}}_{\text{Zero State}} + \underbrace{\frac{GLi_L(0) + (GLs + 1)Cv_C(0)}{GLCs^2 + Cs + G}}_{\text{Zero input}}$$

Zero State Zero input

Network Functions

$$\text{Network function} = \frac{\text{Zero-state Response Transform}}{\text{Input Signal Transform}}$$

- Driving-point function relates the voltage and current at a given pair of terminals called a port

$$Z(s) = \frac{V(s)}{I(s)} = \frac{1}{Y(s)}$$

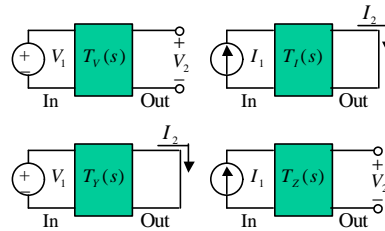
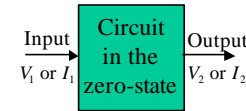
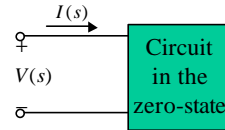
- Transfer function relates an input and response at different ports in the circuit

$$T_V(s) = \text{Voltage Transfer Function} = \frac{V_2(s)}{V_1(s)}$$

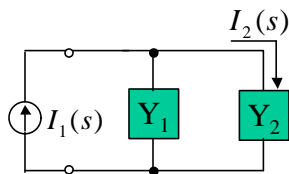
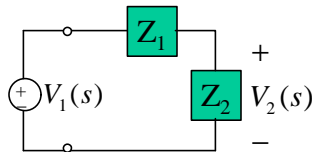
$$T_I(s) = \text{Current Transfer Function} = \frac{I_2(s)}{I_1(s)}$$

$$T_Y(s) = \text{Transfer Admittance} = \frac{I_2(s)}{V_1(s)}$$

$$T_Z(s) = \text{Transfer Impedance} = \frac{V_2(s)}{I_1(s)}$$



Calculating Network Functions



- Driving-point impedance

$$Z_{EQ}(s) = Z_1(s) + Z_2(s)$$

- Voltage transfer function:

$$V_2(s) = \left[\frac{Z_2(s)}{Z_1(s) + Z_2(s)} \right] V_1(s)$$

$$T_V(s) = \frac{V_2(s)}{V_1(s)} = \frac{Z_2(s)}{Z_1(s) + Z_2(s)}$$

- Driving-point admittance

$$Y_{EQ}(s) = Y_1(s) + Y_2(s)$$

- Voltage transfer function:

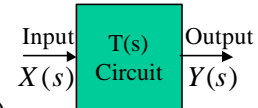
$$I_2(s) = \left[\frac{Y_2(s)}{Y_1(s) + Y_2(s)} \right] I_1(s)$$

$$T_I(s) = \frac{I_2(s)}{I_1(s)} = \frac{Y_2(s)}{Y_1(s) + Y_2(s)}$$

Impulse Response and Step Response

- Input-output relationship in s-domain

$$Y(s) = T(s)X(s)$$



- When input signal is an impulse $x(t) = \mathbf{\delta}(t)$

$$Y(s) = T(s) \times 1 = T(s)$$

- Impulse response equals network function
- $H(s)$ = impulse response transform
- $h(t)$ = impulse response waveform

- When input signal is a step $x(t) = u(t)$

- $G(s)$ = step response transform

- $g(t)$ = step response waveform

$$G(s) = \frac{T(s)}{s} = \frac{H(s)}{s}$$

$$g(s) = \int_0^t h(\mathbf{\tau})d\mathbf{\tau}, \quad h(t)(=) \frac{dg(t)}{dt}$$

(=) means equal almost everywhere, excludes those points at which $g(t)$ has a discontinuity