Stochastic Capacitance Extraction Considering Process Variation

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Outline

- **Motivation**
  - Capacitance extraction
  - Process variations

- **Basic Capacitance Extraction [1]**

- **Statistical Capacitance Extraction**
  - Principle Factor Analysis based method [2]
  - Orthogonal Polynomial method [4]

- **Experiment Results**

- **Other works [3,5] and Future Research**
Importance of Capacitance Extraction

- **Static Timing Analysis**
  - Gate delay ↓
  - Interconnect delay ↑

- **Signal Integrity Analysis**
  - Ground capacitance ↓
  - Coupling capacitance ↑

- **Design Verification**
  - Timing convergence
Capacitance Extraction Issues

- Efficiency requirement
  - Slow large-scale dense matrix solver
  - Prohibitive memory consumption

- Accuracy requirement
  - Single line capacitance
  - Trade off between accuracy and efficiency

- Process variations induce capacitance variation
  - Interconnect surface fluctuation
  - Compatibility with timing analysis engine
Process Variations

- Chemical-mechanical planarization

- Chemical etching

- Optical proximity effects
Statistic Capacitance Extraction Issues

- How to model conductor surface fluctuation due to process variations?
- How to efficiently solve capacitance variation in terms of surface variations?
- How to achieve compatibility with model order reduction and statistic timing analysis engines?
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- Other works and Future Research
3D Capacitance Extraction

From $Q = Cv$

$C_{i,j} = -Q_j = -\sum_{n=1}^{2} q_{j,n}$
3D Capacitance Extraction (cont.)

- Matrix inversion is $n^3$

$$
\Phi(r) = \frac{1}{4\pi\varepsilon} \int \frac{\rho(r')}{\|r - r'\|} ds'
$$

*If there are $n$ charges: $q_i$*

$$
\Phi = \sum_{i=1}^{n} P_{ij} q_i
$$

\[
\begin{bmatrix}
P_{i,1}^{i,1} & P_{i,1}^{i,2} & P_{j,1}^{i,1} & P_{j,2}^{i,1} \\
P_{1,1}^{i,2} & P_{1,2}^{i,2} & P_{j,1}^{i,2} & P_{j,2}^{i,2} \\
P_{1,1}^{j,1} & P_{1,2}^{j,1} & P_{j,1}^{j,1} & P_{j,2}^{j,1} \\
P_{1,1}^{j,2} & P_{1,2}^{j,2} & P_{j,1}^{j,2} & P_{j,2}^{j,2}
\end{bmatrix}
\begin{bmatrix}
q_{i,1} \\
q_{i,2} \\
q_{j,1} \\
q_{j,2}
\end{bmatrix}
= \begin{bmatrix}
1 \\
1 \\
0 \\
0
\end{bmatrix}
\]
Hierarchical & Multipole Speed-up

- Hierarchical refinement and Multipole Expansion

- Far field interaction

- Near field interaction
Hierarchical Panel Refinement

(Hicap weping shi, et al)

\[
H = \begin{bmatrix}
a & b & c & d & e & f & g & h & i & j \\
ab & cd & ef & gh & hi & ij \\
a & b & c & d & e & f & g & h & i & j \\
\end{bmatrix}
\]

\[
J = \begin{bmatrix}
d & f & g & h & i & j \\
1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 \\
\end{bmatrix}
\]

\[
P = J^T H J
\]
ICCAP – Explicit Sparse Matrix with $O(n)$ Sparsification

Every panel charge can be represented as an unique linear combination of charges on basis panels.
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  - Orthogonal Polynomial method

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- Conclusion and Future Research
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Principle Factor Analysis based Stochastic Capacitance Extraction Flow

- Stochastic Capacitance Extraction
- Surface variation modeling
- Random variable reduction
- Random variable matching

\[ \Delta \tilde{n} = \text{leaf panel variance} \]
\[ \Sigma(\Delta \tilde{n}) = \text{Covariance matrix} \]

\[ \Delta \tilde{n} = L \delta. \text{ Principle Factor Analysis} \]
Variable #: \( m \Rightarrow n \) after PFA

\[ F + \sum F_k \delta_k + \sum F_{kl} \delta_k \delta_l \]
For each leaf panel $i$, there is a perturbation $\tilde{\Delta}n_i$ along its normal direction.

Assume that the perturbation is described by a Gaussian random variable $\tilde{\Delta}n_i$ with mean $\mu_i = 0$ and standard deviation $\sigma_i$. 
Variance and Covariance Matrix

- The correlation between leaf panel i and leaf panel j is determined by Gaussian correlation function:
  \[ g(r_{ij}) = \exp\left(-\frac{r_{ij}^2}{\rho^2}\right), \]
- The correlation matrix \( \mathbf{\Gamma}(\Delta n) \sim = \begin{bmatrix} g(r_{ij}) \end{bmatrix} \).
- The standard deviations on panel i and panel j are \( s_i \) and \( s_j \), then the variance and covariance matrix of leaf panels will be \( \mathbf{\Sigma}(\Delta n) = \begin{bmatrix} g(r_{ij}) \sigma_i \sigma_j \end{bmatrix} \).
Principle Factor Analysis

- Represent leaf panel variations in terms of another set of random variables \( \delta \). 
  \[ \Delta \tilde{n} = L' \delta \]

- Random variables \( \delta \) are of normal distribution and are independent (orthogonal).
  \[ \Sigma(\delta) = I, \quad \mu(\delta) = 0 \]

\[ \begin{align*}
\Sigma(\Delta \tilde{n}) &= \Sigma(L\delta) = L \Sigma(\delta)L' = LL' \\
\Sigma(\Delta \tilde{n}) &= \lambda_1 e_1 e_1' + \lambda_2 e_2 e_2' + \cdots + \lambda_m e_m e_m' + \cdots + \lambda_n e_n e_n' \\
&\approx \begin{bmatrix} \sqrt{\lambda_1} e_1' \\ \vdots \\ \sqrt{\lambda_m} e_m' \end{bmatrix} \begin{bmatrix} \sqrt{\lambda_1} e_1 \\ \vdots \\ \sqrt{\lambda_m} e_m \end{bmatrix}
\end{align*} \]
Potential Coefficient Matrix

\[ P_{ij} \propto \frac{1}{\sqrt{(x_i-x_j)^2 + (y_i-y_j)^2 + (z_i-z_j)^2}} \]

\[ \tilde{P}_{ij} \propto \frac{1}{\sqrt{(x_i+\Delta n_i-x_j-\Delta n_j)^2 + (y_i-y_j)^2 + (z_i-z_j)^2}} \]

\[ \approx P_{ij} + a \Delta n + \Delta n' A \Delta n \quad \text{quadratic form} \]

\[ = P_{ij} + a \begin{bmatrix} R_i \\ R_j \end{bmatrix} \delta + \delta' \begin{bmatrix} R_i \\ R_j \end{bmatrix}' A \begin{bmatrix} R_i \\ R_j \end{bmatrix} \delta \]

\[ = \Delta n = \begin{bmatrix} \Delta n_i \\ \Delta n_j \end{bmatrix} = \begin{bmatrix} R_i \\ R_j \end{bmatrix} \delta \]
Problem Definition

- **Known:**
  - Nominal linear system: \( Pq = \nu \)
  - Variational linear system: \( \tilde{P}\tilde{q} = \nu \)
  - Variational potential coefficient matrix:

\[
\tilde{P} = P + \sum P_k \delta_k + \sum P_{kl} \delta_k \delta_l
\]

- **Unknown:**
  - Represent \( \tilde{q} \) in quadratic format:

\[
\tilde{q} = q + \sum q_k \delta_k + \sum q_{kl} \delta_k \delta_l
\]
Random Variable Matching

Normal linear system: \( Pq = \nu \)

Variational linear system: \( \tilde{P}q = \nu \)

\[
\begin{aligned}
\tilde{P} &= P + \sum P_k \delta_k + \sum P_{kl} \delta_k \delta_l \\
\tilde{q} &= q + \sum q_k \delta_k + \sum q_{kl} \delta_k \delta_l
\end{aligned}
\]

unknowns

\[
( P + \sum P_k \delta_k + \sum P_{kl} \delta_k \delta_l ) ( q + \sum q_k \delta_k + \sum q_{kl} \delta_k \delta_l ) = V
\]

\[
\begin{aligned}
P q_k &= -P_k q \quad k=1,2,...,m \\
P q_{kk} &= -(P_{kk} q + P_k q_k) \quad k=1,2,...,m \\
P q_{kl} &= -(P_{kl} q + P_k q_l + P_l q_k) \quad k=1,2,...,m \quad l=1,2,...,m \quad k \neq l
\end{aligned}
\]
## Experimental Results (PFA)

### 2 x 2 bus

<table>
<thead>
<tr>
<th>Monte Carlo</th>
<th>PFA</th>
<th>Speed up/ accuracy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Run time (s)</td>
<td>1826</td>
<td>9.78</td>
</tr>
<tr>
<td>Mean</td>
<td>78.56</td>
<td>81.43</td>
</tr>
<tr>
<td>Variance</td>
<td>106.01</td>
<td>103.64</td>
</tr>
</tbody>
</table>
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Orthogonal Polynomial Method

- Background: orthogonal polynomial (OP) based stochastic analysis method.

- Given: a random variable $v(t, \xi)$ with variation, where $\xi = [\xi_1, ..., \xi_n]$ denotes a vector of orthonormal Gaussian random variables.

\[
\begin{align*}
\text{truncated Hermite Polynomial expansion} & \quad \Rightarrow \\
& \quad v(t, \xi) = \sum_{k=0}^{P} a_k H_k^n(\xi)
\end{align*}
\]

- $n$ is the number of independent random variables.
- $H_k^n(\xi)$ are Hermite polynomials, or orthogonal basis.
- $a_k$ are the expansion coefficients.
- $p$ is the order of the Hermite Polynomial Expansion.
Orthogonal Polynomial Method (cont.)

- Orthogonal Property (inner product)

\[ \langle H_i(\xi), H_j(\xi) \rangle = \langle H_i^2(\xi) \rangle \delta_{ij} \]

- \( \langle *, * \rangle \) denotes an inner product defined as:

\[ \langle f(\xi), g(\xi) \rangle = \frac{1}{\sqrt{(2\pi)^n}} \int f(\xi)g(\xi)e^{-\frac{1}{2}\xi^T\xi}d\xi. \]

- With orthogonal property, \( a_k \) can be evaluated by projection onto Hermiet Polynomial basis:

\[ a_k(t) = \frac{\langle \nu(t, \xi), H_k(\xi) \rangle}{\langle H_k^2(\xi) \rangle}, \ \forall k \in \{0, \ldots, P\} \]
Orthogonal Polynomial Method
Process Variation Modeling

- For each leaf panel \( i \), there is a perturbation \( \Delta n_i \) along its normal direction.

- The potential coefficient matrix \( P \) can be:

\[
P_{ij} \approx G(x_i, x_j) \quad i \neq j
\]

\[
G(x_i, x_j) = \frac{1}{|x_i - x_j|}
\]

- Due to orthogonal property, we can express \( P_{ij} \) with orthogonal basis \( H_k(\xi) \)

\[
P_{ij}(\xi) = \sum_{k=1}^{n} a_k H_k(\xi); \quad a_k = \frac{\langle P_{ij}(\xi), H_k(\xi) \rangle}{\langle H_k^2(\xi) \rangle}, \quad \forall k \in \{0, \ldots, n\}
\]
Orthogonal Polynomial Method based Stochastic Capacitance Extraction

Given Problem:
- Nominal linear system: \( Pq = v \)
- Variational linear system: \( P(\xi)q(\xi) = v \)
- Potential coefficient matrix:
  \[
P(\xi) = P_0 + P_1(\xi) = P_0 + \sum_{i=1}^{p} P_i H_i(\xi)
\]

Solve:
- Represent \( q(\xi) \) with the same orthogonal basis:
  \[
  q(\xi) = q_0 + \sum_{i=1}^{K} q_i H_i(\xi)
  \]
OPM based Stochastic Capacitance Extraction Flow

1. OPM Stochastic Capacitance Extraction
   - Surface variation modeling for $P(\xi)$

2. Solve Variational Linear System for $q(\xi)$

3. Compute Capacitance with solved charge

Mathematical Formulas:

- $P(\xi) = \sum_{k=1}^{n} a_k H_k(\xi)$

- $P(\xi)q(\xi) = \nu \rightarrow q(\xi) = q_0 + \sum_{k=1}^{n} q_k H_k(\xi)$

- $C_{i,j} = -Q_j = -\sum_{n=1}^{m} q_{j,n}$
OPM – solve variational linear system

- From variation modeling, we get potential coefficient equation as:

\[ P(\xi)q(\xi) = \nu \quad \Rightarrow \quad Pq = (P_0 + \sum_{i=1}^{p} P_{1i}H_i)(q_0 + \sum_{i=1}^{k} q_iH_i) = \nu \]

- With principle of orthogonality, we can perform inner product with \( H_i \) on both sides:

\[ <P(\xi)q(\xi), H_k(\xi)> = <\nu, H_k(\xi)> \]

Then we derive a new linear system equation:

\[ W*Q = V \]

\[ Q = \begin{pmatrix} q_0 \\ q_1 \\ \vdots \\ q_K \end{pmatrix} ; \quad V = \begin{pmatrix} \nu \\ 0 \\ \vdots \\ 0 \end{pmatrix} \]
## OPM Experiment Result (1)

### Runtime Comparison

<table>
<thead>
<tr>
<th>Configuration</th>
<th>MC (s)</th>
<th>OPM (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1x1 bus, MC (10000)</td>
<td>9414</td>
<td>3.2</td>
</tr>
<tr>
<td>2X2 Bus, MC (6000)</td>
<td>286620</td>
<td>268</td>
</tr>
<tr>
<td>3-layer metal plane, MC (2000)</td>
<td>5479</td>
<td>7.6</td>
</tr>
</tbody>
</table>
Accuracy Comparison (mean value)

<table>
<thead>
<tr>
<th></th>
<th>MC</th>
<th>OPM</th>
</tr>
</thead>
<tbody>
<tr>
<td>C11</td>
<td>242.54</td>
<td>242.63</td>
</tr>
<tr>
<td>C12</td>
<td>-82.46</td>
<td>-82.60</td>
</tr>
<tr>
<td>C13</td>
<td>-47.48</td>
<td>-47.34</td>
</tr>
<tr>
<td>C14</td>
<td>-47.42</td>
<td>-47.35</td>
</tr>
<tr>
<td>C22</td>
<td>242.34</td>
<td>242.65</td>
</tr>
<tr>
<td>C23</td>
<td>-47.31</td>
<td>-47.36</td>
</tr>
<tr>
<td>C24</td>
<td>-47.25</td>
<td>-47.36</td>
</tr>
<tr>
<td>C33</td>
<td>242.52</td>
<td>242.64</td>
</tr>
<tr>
<td>C34</td>
<td>-82.50</td>
<td>-82.60</td>
</tr>
<tr>
<td>C44</td>
<td>242.47</td>
<td>242.64</td>
</tr>
</tbody>
</table>

2x2 Bus, Panel num=352, $\sigma = 0.1$, $\eta = 2$
Other methods

- Discretize conductor surface based on correlation function to reduce random variable. [3]
- Use window technique for full-chip capacitance extraction. [5]

Possible improvement

- Adaptive parallel extraction using GPU.


Thank you & Questions?
Correlation function

\[ \text{Cov}(\vec{r}_i, \vec{r}_j) = \sigma^2 \exp\left(-\frac{|\vec{r}_i - \vec{r}_j|^2}{\eta^2}\right) \]

where \( \sigma \) is the standard variance and \( \eta \) is the correlation length.

\[ \int_{\Omega} \text{Cov}(\vec{r}_1, \vec{r}_2) g_n(\vec{r}_2) d\vec{r}_2 = \lambda_n g_n(\vec{r}_1) \]