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Local Search Strategies for Satisfiability Testing

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ABSTRACT. It has recently been shown that local search is surprisingly good at finding satisfying assignments for certain classes of CNF formulas [**24**]. In this paper we demonstrate that the power of local search for satisfiability testing can be further enhanced by employing a new strategy, called "mixed random walk", for escaping from local minima. We present experimental results showing how this strategy allows us to handle formulas that are substantially larger than those that can be solved with basic local search. We also present a detailed comparison of our random walk strategy with simulated annealing. Our results show that mixed random walk is the superior strategy on several classes of computationally difficult problem instances. Finally, we present results demonstrating the effectiveness of local search with walk for solving circuit synthesis and diagnosis problems.

1. Introduction

Local search algorithms have been successfully applied to many optimization problems. Hansen and Jaumard [**9**] describe experiments using local search for MAX-SAT, *i.e.*, the problem of finding an assignment that satisfies as many clauses as possible of a given conjunctive normal form (CNF) formula. In general, such local search algorithms find good but non-optimal solutions, and thus such algorithms were believed not to be suitable for satisfiability testing, where the objective is to find an assignment that satisfies *all* clauses (if such an assignment exists).[1]

Recently, however, local search has been shown to be surprisingly good at finding completely satisfying assignments for CNF problems [**24, 8**]. Such methods outperform the best known systematic search algorithms on certain classes of large satisfiability problems. For example, GSAT, a randomized local search

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[1]A clause is a disjunction of literals. A literal is a propositional variable or its negation. A set of clauses corresponds to a CNF formula: a conjunction of disjunctions.

algorithm, can find satisfying assignments of computationally hard randomly-generated 3CNF formulas with over 2000 variables, whereas the current fastest systematic search algorithms cannot handle instances from the same distribution with more than 400 variables [**1, 4**].

We should stress that local search methods for satisfiability testing are inherently incomplete. That is, when a formula is satisfiable, these methods can often find a satisfying assignment, but they cannot show that no such assignment exists. This has led to an interesting shift in the way problems are encoded as satisfiability problems. Traditionally, in the area of artificial intelligence, many tasks were formulated as theorem proving problems, and, therefore, much research focussed on methods for showing inconsistency or unsatisfiability. The recent success of incomplete methods has led to the reformulation of some of these problems into model-finding tasks, where solutions correspond to satisfying assignments of the SAT encodings. Incomplete methods can then be used to search for such assignments [**15, 6**]. When we say that an incomplete method can solve much larger instances of a certain problem class than complete methods, we are referring to its ability to find satisfying assignments (*i.e.*, models) of satisfiable instances in the problem class.

The basic GSAT algorithm performs a local search of the space of truth-assignments by starting with a randomly-generated assignment, and then repeatedly changing ("flipping") the assignment of a variable that leads to the largest decrease in the total number of unsatisfied clauses. As with any combinatorial problem, local minima in the search space are problematic in the application of local search methods. A local minimum is defined as a state whose local neighborhood does not include a state that is strictly better. The standard approach in combinatorial optimization of terminating the search when a local minimum is reached [**21**] does not work well for Boolean satisfiability testing, since only global optima are of interest. In Selman *et al.* [**24**], it is shown that simply continuing to search by making non-improving, "sideways" moves, dramatically increases the success rate of the algorithm. Figure 1 illustrates a typical search, plotting the number of unsatisfied clauses as a function of the number of flips performed. From the figure it is clear that the search begins with a rapid greedy descent followed by a long sequences of sideways moves. We refer to each sequence of sideways moves as a *plateau*. Note that actual uphill moves almost never occur. For a detailed quantitative study of the search space, see [**7**].

The success of GSAT is determined by its ability to move between successively lower plateaus. The search fails if GSAT can find no way off of a plateau, either because such transitions from the plateau are rare or nonexistent. When this occurs, one can simply restart the search at new random initial assignment. There are other mechanisms for escaping from local minima, which are based on occasionally making uphill moves. Prominent among such approaches has been the use of simulated annealing [**16**], where a formal parameter (the "temperature") controls the probability that the local search algorithm makes an uphill

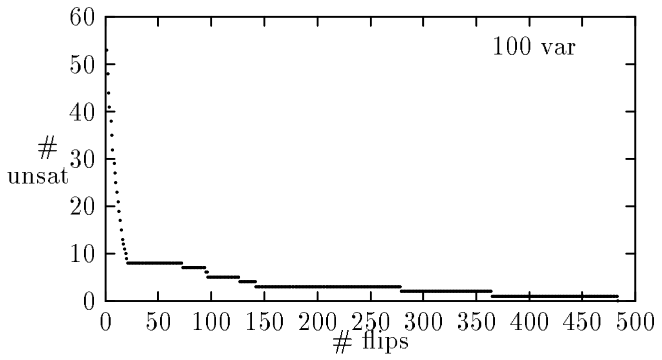

FIGURE 1. GSAT's search space on a randomly-generated 100 variable 3CNF formula with 430 clauses.

move.

We propose a new mechanism for introducing such uphill moves, and provide experimental evidence that our method outperforms simulated annealing on several classes of hard Boolean satisfiability problems. The strategy is based on mixing a random walk over variables that appear in unsatisfied clauses with the greedy local search. The strategy can be viewed as a way of introducing noise in a very focused manner — namely, perturbing only those variables critical to to the remaining unsatisfied clauses.

We will present experimental data comparing our random walk strategy, simulated annealing, random noise, and the basic GSAT procedure on computationally difficult random formulas. In doing this comparison, we tuned the parameter settings of each procedure to obtain their best performance. We will see that the random walk strategy significantly outperforms the other approaches, and that all the escape strategies are an improvement over basic GSAT.

One might speculate that the good performance of the random walk strategy is a consequence of our choice of test instances. We therefore also ran experiments using several other classes of problem instances, developed by transforming other combinatorial problems into satisfiability instances. In particular, we considered problems from planning [**15**] and circuit synthesis [**13**]. These experiments again demonstrate that mixed random walk is the superior escape mechanism. In addition, we show that GSAT with walk is faster than systematic search on certain circuit synthesis problems (such as adders and comparators) that contain *no* random component. Finally, we present data on experiments with a modified version of the random walk strategy that further improves performance over GSAT with walk.

2. Local Search for Satisfiability Testing

GSAT performs a greedy local search for a satisfying assignment of a set of

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Procedure GSAT
for $i$ := 1 to MAX-TRIES
    $T$ := a randomly generated truth assignment
    for $j$ := 1 to MAX-FLIPS
        if $T$ satisfies $\alpha$ then return $T$
        Flip any variable in $T$ that results in greatest
             decrease (can be 0 or negative)
             in the number of unsatisfied clauses
    end for
end for
return "No satisfying assignment found"
```

FIGURE 2. The GSAT procedure.

propositional clauses. See Figure 2. The procedure starts with a randomly generated truth assignment. It then changes ('flips') the assignment of the variable that leads to the greatest decrease in the total number of unsatisfied clauses. Such flips are repeated until either a satisfying assignment is found or a pre-set maximum number of flips (MAX-FLIPS) is reached. This process is repeated as needed up to a maximum of MAX-TRIES times.

In [**24**], we showed that GSAT substantially outperforms backtracking search procedures, such as the Davis-Putnam procedure, on various classes of formulas, including randomly generated formulas and SAT encodings of graph coloring problems [**12**].

As noted above, local minima in the search space of a combinatorial problem are the primary obstacle to the application of local search methods. GSAT's use of sideways moves does not completely eliminate this problem, because the algorithm can still become stuck on a plateau (a set of neighboring states each with an equal number of unsatisfied clauses). Therefore, it is useful to employ mechanisms that escape from local minima or plateaus by making uphill moves (flips that increase the number of unsatisfied clauses). We will now discuss two mechanisms for making such moves.[2]

2.1. Simulated Annealing. Simulated annealing introduces uphill moves into local search by using a noise model based on statistical mechanics [**16**]. We employ the annealing algorithm defined in [**12**]: Start with a randomly generated truth assignment. Repeatedly pick a random variable, and compute δ, the change in the number of unsatisfied clauses when that variable is flipped. If $\delta \leq 0$ (a downhill or sideways move), make the flip. Otherwise, flip the variable with probability $e^{-\delta/T}$, where T is a formal parameter called the *temperature*.

[2]If the only possible move for GSAT is uphill, it will make such a move, but such "forced" uphill moves are quite rare, and are not effective in escaping from local minima or plateaus.

The temperature may be either held constant,[3] or slowly decreased from a high temperature to near zero according to a cooling schedule. One often uses *geometric* schedules, in which the temperature is repeatedly reduced by multiplying it by a constant factor (< 1).

Given a finite cooling schedule, simulated annealing is not guaranteed to find a global optimum — that is, an assignment that satisfies all clauses. Therefore in our experiments we use multiple random starts.

The basic GSAT algorithm is very similar to annealing at temperature zero, but differs in that GSAT naturally employ restarts and *always* makes a downhill move if one is available. required to find a solution.

2.2. The Random Walk Strategy. Consider the following algorithm for testing the satisfiability of CNF formulas. Start with a random truth assignment; randomly select a clause not satisfied by this assignment; flip the truth assignment of one of the letters occuring in this clause (the clause becomes satisfied); repeat the last two steps until the assignment satisfies all clauses. Papadimitriou [**20**] shows that such a surprisingly simple randomized strategy finds assignments for 2CNF formulas in $O(n^2)$ steps with probability approaching one, where n is the number of propositional letters. However, this method does not work for general CNF.

We therefore propose the following approach, that mixes the 2CNF strategy with greedy search:

```
Random Walk Strategy
With probability $p$, pick a variable occuring in some
        unsatisfied clause and flip its truth assignment.
With probability $1 - p$, follow the standard GSAT scheme,
        i.e., make the best possible local move.
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A natural and simpler variation of the random walk strategy is not to restrict the choice of a randomly flipped variable to the set of variables that appear in unsatisfied clauses. We will refer to this modification as the *random noise* strategy. Note that random walk differs from both simulated annealing and random noise, in that in random walk upward moves are closely linked to unsatisfied clauses. The experiments discussed below will show that the random walk strategy is generally significantly better.

3. Experimental Results

We compared the basic GSAT algorithm, simulated annealing, random walk, and random noise strategies on a test suite including both randomly-generated CNF problems and Boolean encodings of other combinatorial problems. The results are given in the Tables 1, 2, and 3. For each strategy we give the average

[3]This form of annealing corresponds to the Metropolis algorithm [**10**].

time in seconds it took to find a satisfying assignment and the average number of flips it required.

For each strategy we used at least 100 random restarts (MAX-TRIES setting in GSAT) on each problem instance; if we needed more than 20 restarts before finding an assignment, then the strategy was restarted up to 1,000 times. With this choice of the maximum number of restarts, almost all averages in the tables are based on at least five successful runs. A "*" in the tables indicates that no solution was found after running for more than 20 hours or using more than 1,000 restarts.[4]

The parameters of each method were varied over a range of values, and only the results of the best settings are included in the table. For basic GSAT, we varied MAX-FLIPS and MAX-TRIES; for GSAT with random walk, we also varied the probability p with which a non-greedy move is made, and similarly for GSAT with random noise. In all of our experiments, the optimal value of p was found to be between 0.5 and 0.6. For constant temperature simulated annealing, we varied the temperature T from 5 to 0 in steps of 0.05. (At $T = 5$, uphill moves are accepted with probability greater than 0.8.) For the random formulas, the best performance was found at $T = 0.2$. The planning formulas required a higher temperature, $T = 0.5$, while the Boolean circuit synthesis problems were solved most quickly at a low temperature, $T = 0.15$.

We also experimented with various geometric cooling schedules. Surprisingly, we did not find any geometric schedule that was better than the best constant-temperature schedule. We could not even significantly improve the average number of restarts needed before finding a solution by extremely slow cooling schedules, regardless of the effect on execution time. A possible explanation for this is that almost all the work in solving CNF problems lies in satisfying the last few unsatisfied clauses. This corresponds to the low-temperature tail of a geometric schedule, where the temperature has little variation.

3.1. Hard Random Formulas. Random instances of CNF formulas are often used in evaluating satisfiability procedures because they can be easily generated and lack any underlying "hidden" structure often present in hand-crafted instances. Unfortunately, unless great care is taken in specifying the parameters of the random distribution, the problems so created can be trivial to solve; for example, an algorithm that simply guessed a random truth assignment might be very likely to succeed [**5**]. Mitchell *et al.* [**19**] demonstrate that computationally difficult random problems can be generated using the uniform distribution or fixed-clause length model as follows: Let N be the number of variables, K the number of literals per clause, and L the number of clauses. Each instance is obtained by generating L random clauses each containing K literals. The K literals are generated by randomly selecting K variables, and each of the vari-

[4]The algorithms were implemented in C and ran on an SGI Challenge with a 100 MHz MIPS R4400 processor. For code and experimental data, contact the first author.

formula		GSAT						Simul. Ann.	
		basic		walk		noise			
vars	clauses	time	flips	time	flips	time	flips	time	flips
100	430	.4	7554	.2	2385	.6	9975	.6	4748
200	860	22	284693	4	27654	47	396534	21	106643
400	1700	122	2.6×10^6	7	59744	95	892048	75	552433
600	2550	1471	30×10^6	35	241651	929	7.8×10^6	427	2.7×10^6
800	3400	*	*	286	1.8×10^6	*	*	*	*
1000	4250	*	*	1095	5.8×10^6	*	*	*	*
2000	8480	*	*	3255	23×10^6	*	*	*	*

TABLE 1. Comparing noise strategies on hard random 3CNF instances.

ables is negated with a 50% probability. The difficulty of such formulas critically depends on the ratio between N and L. The hardest formulas lie around the region where there is a 50% chance of the randomly generated formula being satisfiable. For 3CNF formulas ($K = 3$), experiments show that this is the case for $L \approx 4.3N$. (For larger N the the critical ratio for the 50% point converges to approximately 4.2 [**2, 17**].) We tested the algorithms on formulas around the 4.3 point ranging in size from 100 to 2000 variables.

Table 1 presents our results. For the smallest (100-variable) formula, we observe little difference in the running times. As the number of variables increase, however, the random walk strategy significantly dominates the other approaches. Both random noise and simulated annealing also improve upon basic GSAT, but neither of these methods found solutions for largest three formulas.[5] The performance of GSAT with walk is quite impressive, especially considered the fact that fastest current systematic search methods cannot find satisfying assignments for hard random 3CNF instances with over 400 variables [**4**].

The columns marked with "flips" give the average number of flips required to find an assignment. (A "flip" in our simulated annealing algorithm is an actual change in the truth assignment. We do not count flips that were considered but not made.) When comparing the number of flips required by the various strategies, we arrive at the same conclusion about the relative efficiencies of the methods. This shows that our observations based on the running times are not simply a consequence of differences in the relative efficiencies of our implementations.

Finally, let us briefly consider the average number of restarts needed before finding a solution. Basic GSAT easily gets stuck on plateaus, and requires many random restarts, in particular for larger formulas. On the other hand, our experiments show that GSAT with walk is practically guaranteed to find a satisfying

[5]GSAT with walk finds approximately 50% of the formulas in the hard region to be satisfiable, as would be expected at the transition point for SAT. For example, we considered a set of 1000 variable, 4246 clause formulas (a good estimate of the 50% point). Out of 40 formulas, we found 24 to be satisfiable.

formula			GSAT						Simul. Ann.	
			basic		walk		noise			
id	vars	clauses	time	flips	time	flips	time	flips	time	flips
med.	273	2311	7.5	70652	0.4	3464	4.5	41325	4.5	12147
rev.	201	1382	3.7	41693	0.3	3026	2.5	29007	2.7	9758
hanoi	417	2559	*	*	46	334096	1825	16×10^6	2790	4.1×10^6
huge	937	14519	*	*	38	143956	5340	37×10^6	7161	4.4×10^6

TABLE 2. Comparing noise strategies on SAT encodings of planning problems.

assignment (if one exists). Apparently, mixing random walk over variables in the unsatisfied clauses with greedy moves allows one to escape almost always from plateaus that have few or no states from which a downhill move can be made. The other two strategies also need fewer restarts than basic GSAT but the effect is less dramatic.

3.2. Planning Problems. As a second example of the effectiveness of the various escape strategies, we consider encodings of blocks-world planning problems [**15**]. Such formulas are very challenging for basic GSAT. Examination of the best assignments found when GSAT fails to find a satisfying assignment indicates that difficulties arise from extremely deep local minima. For example, the planning problem labeled "Hanoi" corresponds to the familiar "towers of Hanoi" puzzle, in which one moves a stack of disks between three pegs while never placing a larger disk on top of a smaller disk. There are many truth assignments that satisfy *nearly* all of the clauses that encode this problem, but that are very different from the correct satisfying assignment; for example, such a near-assignment may correspond to slipping a disk out from the bottom of the stack.

As seen in Table 2, GSAT with random walk is far superior. As before, basic GSAT fails to solve the largest problems. GSAT with walk is about 100 times faster than our best results for simulated annealing on the two largest problems, and over 200 times faster than random noise. The random noise and annealing strategies on the large problems also require many more restarts than the random walk strategy before finding a solution.

3.3. Circuit Synthesis. Kamath *et al.* [**13**] developed a set of SAT encodings of Boolean circuit synthesis problems in order to test a satisfiability procedure based on integer programming. The task under consideration was to derive a logical circuit from its input-output behavior. Selman *et al.* [**24**] showed that basic GSAT was competitive with their integer-programming method. In Table 3, we give our experimental results on five of the hardest instances considered by Kamath *et al.* As is clear from the table, both the random walk and the simulated annealing strategies significantly improve upon GSAT, with random walk being somewhat better than simulated annealing. For comparison,

formula		Int.P.	GSAT						Simul. Ann.	
			basic		walk		noise			
id	vars	time	time	flips	time	flips	time	flips	time	flips
f16a1	1650	2039	114	709895	4.2	3371	708	1025454	25	98105
f16b1	1728	78	452	2870019	24	25529	2199	2872226	22	96612
f16c1	1580	758	3.5	12178	1.6	1545	11	14614	8.4	21222
f16d1	1230	1547	174	872219	6.2	5582	371	387491	8.4	25027
f16e1	1245	2156	1.7	2090	1.8	1468	3	3130	6.3	5867

TABLE 3. Comparing noise strategies on the circuit synthesis problem instances as studied in Kamath *et al.*.

we also included the original timings reported by Kamath *et al.*[6] In this case, the random noise strategy does not lead to an improvement over basic GSAT. In fact mixing in random noise appears to degrade GSAT's performance. Note that the basic GSAT procedure already performs quite well on these formulas, which suggests that they are relatively easy compared to our other benchmark problems.

The instances from Kamath *et al.* [**13**] were derived from randomly wired Boolean circuits. So, although the SAT encodings contain some intricate structure from the underlying Boolean gates, there is still a random aspect to the problem instances. Recently, Kamath *et al.* [**14**] have generalized their approach, to allow for circuits with multiple outputs. Using this formulation, we can encode Boolean circuits that are useful in practical applications. Some examples are adder and comparator circuits. We encoded the I/O behavior of several of such circuits, and used GSAT with walk to solve them. Table 4 shows our results. ("GSAT+w" denotes GSAT with walk. We used $p = 0.5$. We will discuss the "WSAT" column below.) The type of circuit is indicated in the table. For example, every satisfying assignment for the formula 2bitadd_11 corresponds to a design for a 2-bit adder using a PLA (Programmable Logic Array). The suffix "11" indicates that the circuit is constrained to use only 11 AND-gates. We see from the table that GSAT with walk can solve the instances in times that range from less than a second to a few minutes. We also included the timings for the Davis-Putnam (DP) procedure. We used a variant of this procedure developed by Crawford and Auton [**2**]. This procedure is currently one of the fastest complete methods, but it is quite surprising to see that it only solves two of the instances.[7] (A "$\star$" indicates that the method ran for 10 hrs without finding an assignment.) The good performance of GSAT with walk on these problems indicates that local search methods can perform well on structured problems that do not contain any random component.

[6]Kamath *et al.*'s satisfiability procedure ran on a VAX 8700 with code written in FORTRAN and C.

[7]Preliminary experiments indicate that some of these formulas can also be solved by combining DP with multiple starts that randomly permute variables. We thank Jimi Crawford for discussions on this issue.

formula			DP	GSAT+w	WSAT
id	vars	clauses	time	time	time
2bitadd_12	708	1702	*	0.081	0.013
2bitadd_11	649	1562	*	0.058	0.014
3bitadd_32	8704	32316	*	94.1	1.0
3bitadd_31	8432	31310	*	456.6	0.7
2bitcomp_12	300	730	23096	0.009	0.002
2bitcomp_5	125	310	1.4	0.009	0.001

Table 4. Comparing an efficient complete method (DP) with local search strategies on circuit synthesis problems. (Timings in seconds.)

formula			DP	GSAT+w	WSAT
id	vars	clauses	time	time	time
ssa7552-038	1501	3575	7	129	2.3
ssa7552-158	1363	3034	*	90	2
ssa7552-159	1363	3032	*	14	0.8
ssa7552-160	1391	3126	*	18	1.5

Table 5. Comparing DP with local search strategies on circuit diagnosis problems by Larrabee. (Timings in seconds.)

3.4. Circuit Diagnosis. Larrabee [**18**] proposed a translation of the problem of test pattern generation for VLSI circuits into a SAT problem. We compared the performance of GSAT with walk and that of DP on several of Larrabee's formulas. Our results are in table 5.[8] We see that GSAT with walk again works very well, especially compared to DP's systematic search. These results and the ones for circuit synthesis are of particular interest because they involve encodings of problems with clear practical applications, and are not just useful as benchmark problems for testing satisfiability procedures.

4. Modifying the Random Walk Strategy

We have recently begun to experiment with a new algorithm that implements GSAT's random walk strategy with subtle but significant modifications. This new algorithm, called WSAT (for "walk sat"), makes flips by first randomly picking a clause that is not satisfied by the current assignment, and then picking (either at random or according to a greedy heuristic) a variable within that clause to flip. Thus, while GSAT with walk can be viewed as adding "walk" to a greedy algorithm, WSAT can be viewed as adding greediness as a heuristic to random walk. The "WSAT" columns in Tables 4 and 5 shows that WSAT can give a

[8]The table contains some typical satisfiable instances from a collection made available by Allan van Gelder and Yumi Tsuji at the University of California at Irvine.

substantial speed up over GSAT with walk. Whether or not WSAT outperforms GSAT with walk appears to depend on the particular problem class. We are currently studying this further.

One unexpected and interesting observation we have already made is that there can be a great variance between running GSAT with 100% walk (*i.e.,* $p = 1.0$) and running WSAT where variables are picked within an unsatisfied clause at random. At first glance, these options would appear to be identical. However, there is a subtle difference in the probability that a given variable is picked to be flipped. GSAT maintains a list (without duplicates) of the variables that appear in unsatisfied clauses, and picks at random from that list; thus, every variable that appears in an unsatisfied clause is chosen with equal probability. WSAT employs the two-step random process described above (first picking a clause, and then picking a variable), that favors variables that appear in many unsatisfied clauses. For many classes of formulas, the difference does not appear to be significant. However, GSAT with 100% walk does not solve the circuit diagnosis problems, whereas WSAT with random picking can solve all of them.

5. Conclusions

We have introduced a random walk strategy for escaping from local minima in satisfiability problems. The approach introduces perturbations in the current state that are directly relevant to the unsatisfied constraints of the problem. Our experiments show that this strategy significantly outperforms simulated annealing and random noise on several classes of hard satisfiability problems. Both of the latter strategies can make perturbations that are in a sense less focused, in that they may involve variables that do not appear in any unsatisfied clauses. The relative improvement found by using random walk over the other methods increases with increasing problem size. We also showed that GSAT with walk to be remarkably efficient in solving basic circuit synthesis problems. This result is especially interesting because the synthesis problems do not have any random component, and are very hard for systematic methods. In the Appendix, we show that GSAT with walk can also efficiently solve many of the DIMACS Challenge Benchmark Instances. Note that GSAT can handle several of the larger instances in the benchmark that are not solved by any other method. For closely related work, see [**22, 25**]. In summary, our results suggest that local search with the appropriate noise strategy is a remarkably powerful method for solving computationally challenging Boolean satisfiability problems.

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