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Model Reduction of Mechanical Systems

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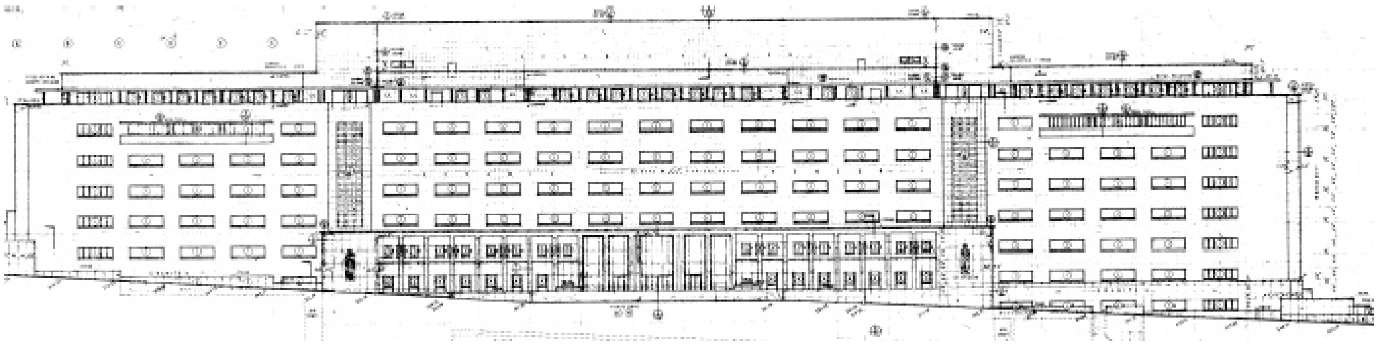
in collaboration with Purdue, Rice, FSU (NSF)

Work Package 2 : Large scale problems and optimization

Research goals or wishful thinking ?



Modeling of mechanical structures

Identification/calibration (cheap sensors)

Simulation/validation (prognosis)

Model reduction

Control (earthquakes, large flexible structures)

Passive / Semi-Active Fluid Dampers

Passive fluid dampers contain bearings and oil absorbing seismic energy.
Semi-active dampers work with variable orifice damping.

(Picture courtesy Steven Williams)

Active Mass Damper

Active Mass Damping via control of displacement, velocity or acceleration of a mass (here by a turn-screw actuator).

Eigenvalue analysis showed dominant transversal mode (0.97 Hz) and torsional mode (1.13Hz). A two-mass active damper damps these modes.

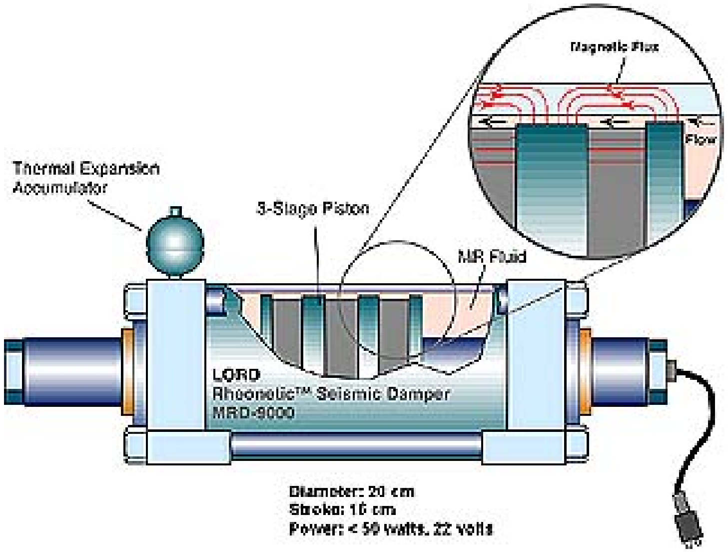
(Picture courtesy Bologna Fiere)

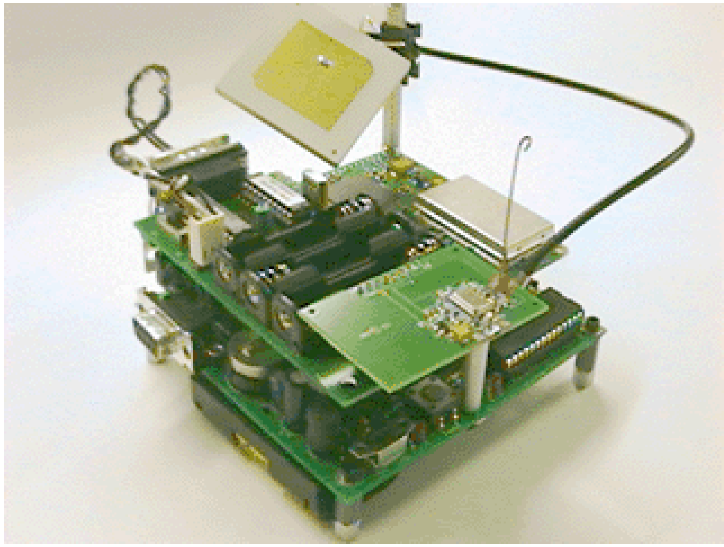
More examples of Control Mechanisms

Building	**Control Mechanism**
CN Tower, Toronto (533m)	Passive Tuned Mass Damper
John Hancock Bldg, Boston (244m)	Two Passive Tuned Dampers
Sydney Tower (305m)	Passive Tuned Pendulum
Rokko Island P&G, Kobe (117m)	Passive Tuned Pendulum
Yokohama Landmark Tower (296m)	Active Tuned Mass Dampers (2)
Shinjuku Park Tower, Tokyo (227m)	Active Tuned Mass Dampers (3)
TYG Building, Atsugi (159m)	Tuned Liquid Dampers (720)

Engineering Structures, Vol. 17, No. 9, Nov. 1995.

The Future: Fine-Grained Semi-Active Control





Dampers are based on Magneto-Rheological fluids with viscosity that changes in milliseconds, when exposed to a magnetic field.

New sensing and networking technology allows to do fine-grained real-time control of structures subjected to winds, earthquakes or hazards.

(Pictures courtesy Lord Corp.)

This technology starts to be applied…

Dongting Lake Bridge has now MR dampers to control wind-induced vibration

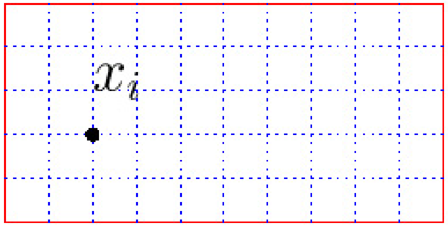
(Pictures courtesy of Prof. Y. L. Xu, Hong Kong Poly.)

Second order system models

Model derived from finite element discretization yield systems of the type

$$M\ddot{x}(t) + D\dot{x}(t) + Sx(t) = Bu(t), \quad y(t) = Cx(t)$$

whose solutions describe “vibrations” in the structure



where

M is the mass matrix ($M = M^T \succ 0$)

S is the stiffness matrix ($S = S^T \succ 0$)

$D(\omega)$ is the damping matrix (frequency dependent)

Reduced order model

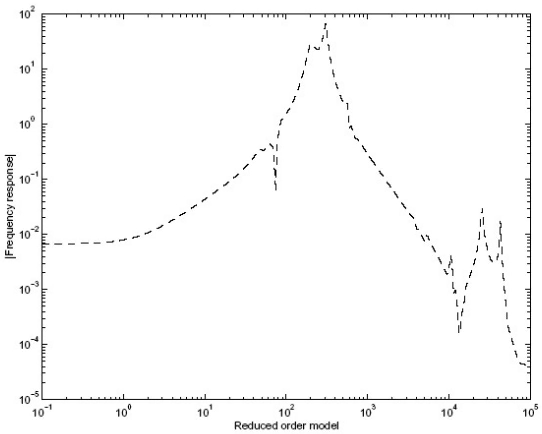
look for smaller model

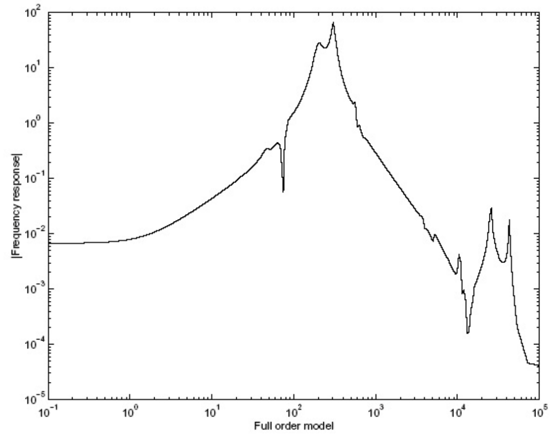
$$\hat{M}\ddot{\hat{x}}(t) + \hat{D}\dot{\hat{x}}(t) + \hat{S}\hat{x}(t) = \hat{B}u(t), \quad \hat{y}(t) = \hat{C}\hat{x}(t)$$

where $u(t) \in \mathbb{R}^m, y(t), \hat{y}(t) \in \mathbb{R}^p,\ x(t) \in \mathbb{R}^N,\ \hat{x}(t) \in \mathbb{R}^n,\ n \ll N$

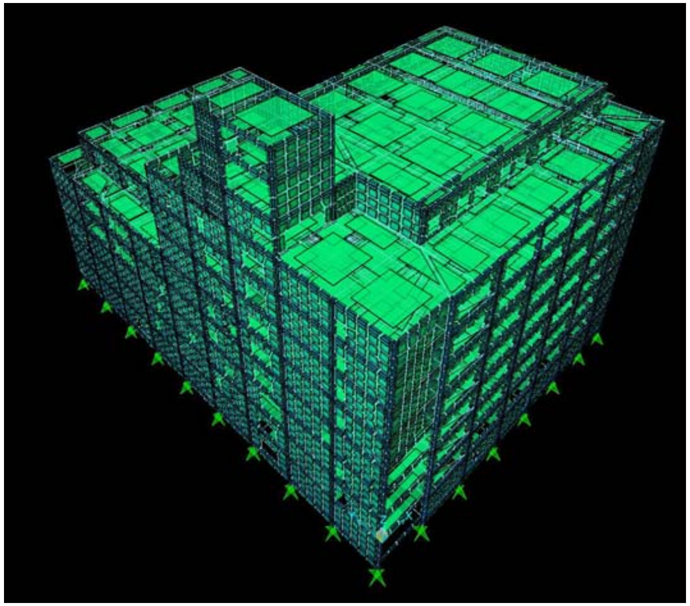
and transfer functions (frequency responses) are close

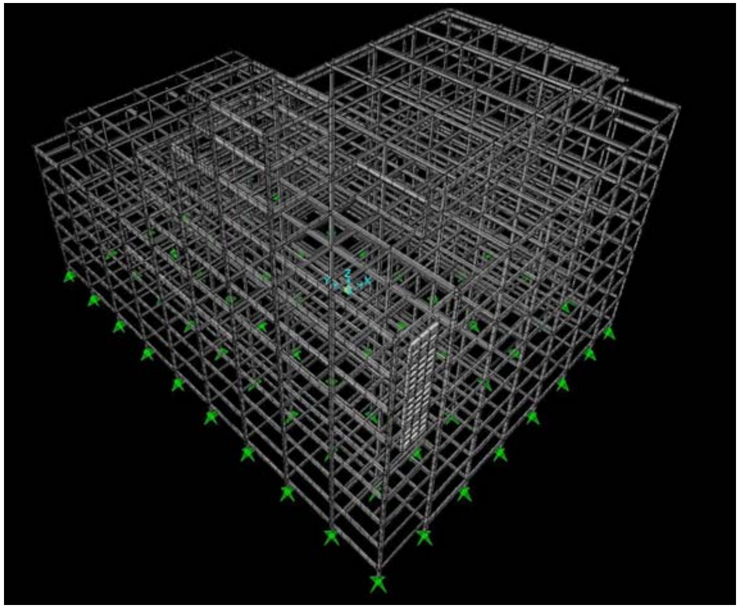
$$\hat{C}(\hat{M}s^2 + \hat{D}s + \hat{S})^{-1}\hat{B} \simeq C(Ms^2 + Ds + S)^{-1}B$$





Start by simplifying the model …

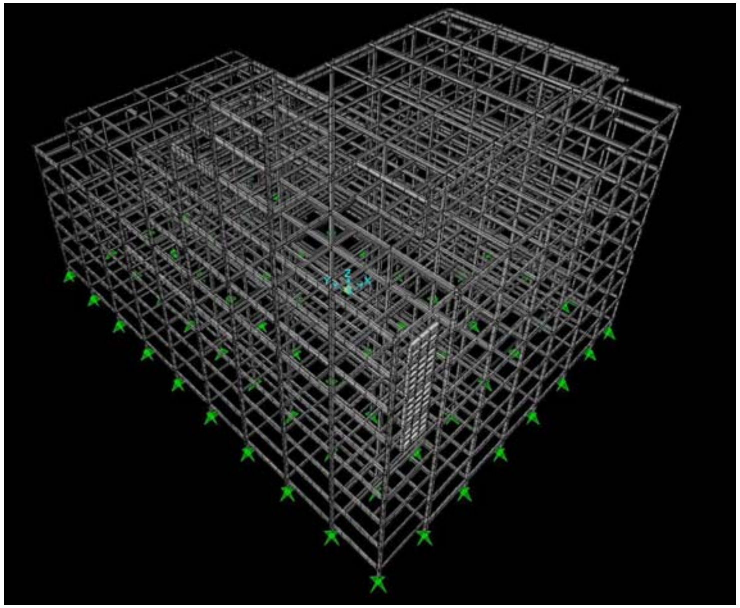




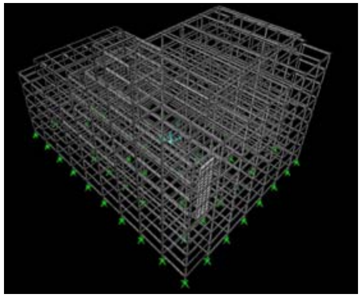
Simplify by
keeping only concrete
substructure

and then reduce the state dimension …

26400 2nd order eqs



20 2nd order eqs



i.e. reduce the number of equations describing the "state" of the system

State space model reduction

Let $V, W \in \mathbb{R}^{N \times n}$, $W^* E V = I_n$ then $\Pi := V W^* E$ is a projector

Choose $W^* E Z = 0_{n,N-n}$ and decompose $x(t) = V\hat{x}(t) + Z\tilde{x}(t)$

Then

$$E\dot{x}(t) = Ax(t) + Bu(t), \quad y(t) = Cx(t)$$

implies

$$\dot{\hat{x}}(t) = W^* A(V\hat{x}(t) + Z\tilde{x}(t)) + W^* Bu(t), \quad y(t) = C(V\hat{x}(t) + Z\tilde{x}(t))$$

This is approximated by the projected system

$$\dot{\hat{x}}(t) = W^* AV\hat{x}(t) + W^* Bu(t), \quad \hat{y}(t) = CV\hat{x}(t).$$

We want a small error $\|y(t) - \hat{y}(t)\|_{\mathcal{L}_2}$ when driven by same input $u(t)$

Gramians measure energy transfer

Minimal $\|y(t) - \hat{y}(t)\|_{\mathcal{L}_2}$ for $\|u(t)\|_{\mathcal{L}_2} = 1$ by minimizing H_∞ norm

$$\sup_\omega \|C(j\omega E - A)^{-1}B - \hat{C}(j\omega I_n - \hat{A})^{-1}\hat{B}\|_2$$

(suboptimal) : Hankel norm approximation and balanced truncation use

$$P = \frac{1}{2\pi}\int_{-\infty}^{\infty}(j\omega E - A)^{-1}BB^T(j\omega E - A)^{-*}d\omega$$

$$Q = \frac{1}{2\pi}\int_{-\infty}^{\infty}(j\omega E - A)^{-*}C^TC(j\omega E - A)^{-1}d\omega$$

Gramians describe how much energy goes through the system via $x(0)$

$$u(-\infty, 0) \longrightarrow x(0) \longrightarrow y(0, \infty)$$

$x(0)^T P^{-1} x(0) = \min \|u(-\infty, 0)\|^2_{\mathcal{L}_2}$ yielding $\|x(0)\|^2_2 = 1$

$x(0)^T E^T Q E x(0) = \|y(0, \infty)\|^2_{\mathcal{L}_2}$ obtained from $\|x(0)\|^2_2 = 1$

Dominant eigenspaces

Use dominant left and right eigenspaces V and W of PE^TQE

- Solve the generalized Lyapunov equations

$$APE^T + EPA^T + BB^T = 0, \quad A^TQE + E^TQA + C^TC = 0$$

- Construct V and W satisfying $W^*EV = I_n$ from

$$\left\{ \begin{array}{rcl} (PE^TQE)V & = & V.\Sigma_+^2 \\ W^*(EPE^TQ) & = & \Sigma_+^2.W^* \end{array} \right. \qquad \left\{ \begin{array}{rcl} QE.V & = & W.\Sigma_+ \\ PE^T.W & = & V.\Sigma_+ \end{array} \right.$$

- The projected system $\{W^*AV, W^*B, CV\}$ has an H_∞ error $\approx \sigma_{n+1}$

Interpolate with Krylov spaces

Let

$$\mathcal{K}_k(M, v) := Im([v, Mv, M^2v, \ldots M^{k-1}v])$$

Choices

$$\bigcup^{i} \mathcal{K}_{k_i}((\sigma_i E - A)^{-1}, (\sigma_i E - A)^{-1}B) \subseteq Im(V)$$

and

$$\bigcup^{j} \mathcal{K}_{\ell_j}((\sigma_j E - A)^{-*}, (\sigma_j E - A)^{-*}C^T) \subseteq Im(W)$$

yield a multipoint Padé approximation

$$C(sE - A)^{-1}B - CV(sI_n - W^*AV)^{-1}W^*B = O(s - \sigma_i)^{k_i}$$

Almost all reduced order systems can be obtained by interpolation

Apply this to 2nd order

Generalized state space model with $\xi^T(t) = [\, x^T(t) \quad \dot{x}^T(t) \,]$

$$\left\{ \begin{array}{rcl} \underbrace{\begin{bmatrix} D & M \\ M & 0 \end{bmatrix}}_{\mathcal{E}} \dot{\xi}(t) & = & \underbrace{\begin{bmatrix} -S & 0 \\ 0 & M \end{bmatrix}}_{\mathcal{A}} \xi(t) + \underbrace{\begin{bmatrix} B \\ 0 \end{bmatrix}}_{\mathcal{B}} u(t), \\ y(t) & = & \underbrace{\begin{bmatrix} C & 0 \end{bmatrix}}_{\mathcal{C}} \xi(t) \end{array} \right.$$

Reduced order model $\{W^*\mathcal{E}V, W^*\mathcal{A}V, W^*\mathcal{B}, \mathcal{C}V\}$ is second order if

$$W = \begin{bmatrix} W_{11} & 0 \\ 0 & W_{22} \end{bmatrix}, \; V = \begin{bmatrix} V_{11} & 0 \\ 0 & V_{22} \end{bmatrix}$$

Structure constraint on projection spaces !

Constrained Gramians

Energy interpretation of diagonal blocks of P, $E^T QE$

$$\min_{u,\dot{x}(0)} \|u(-\infty,0)\|^2_{\mathcal{L}_2} = x(0)^T [P_{11}]^{-1} x(0)$$

$$\min_{u,\dot{x}(0)=0} \|u(-\infty,0)\|^2_{\mathcal{L}_2} = x(0)^T [P^{-1}]_{11} x(0)$$

$$\min_{u,x(0)} \|u(-\infty,0)\|^2_{\mathcal{L}_2} = \dot{x}(0)^T [P_{22}]^{-1} \dot{x}(0)$$

$$\min_{u,x(0)=0} \|u(-\infty,0)\|^2_{\mathcal{L}_2} = \dot{x}(0)^T [P^{-1}]_{22} \dot{x}(0)$$

Similar for Gramians $[E^T QE]_{ii}$ and $[(E^T QE)^{-1}]_{ii}$ using dual system

Use free position and velocity Gramians (CLVV03', Meyer-Srinivasan)

Clamped beam example

n	k	m	p	$\frac{\|\mathcal{H}-\hat{\mathcal{H}}_{BT}\|_2}{\|\mathcal{H}\|_2}$	$\frac{\|\mathcal{H}-\hat{\mathcal{H}}_{SOBT}\|_2}{\|\mathcal{H}\|_2}$
174	17	1	1	2.88e-05	1.83e-04

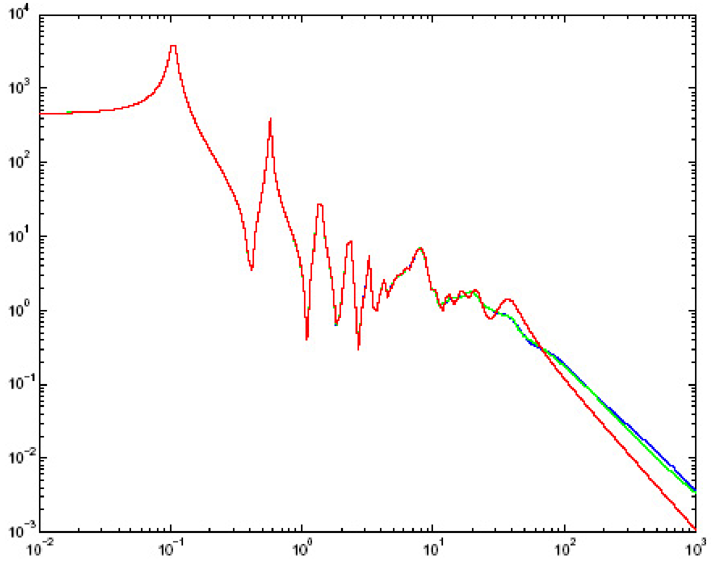
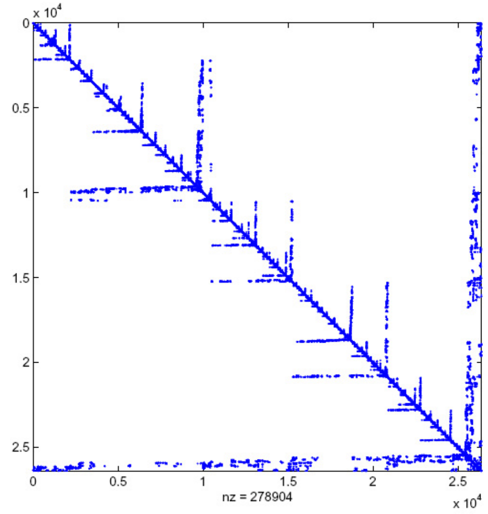


Figure 1: Amplitude of the frequency response.

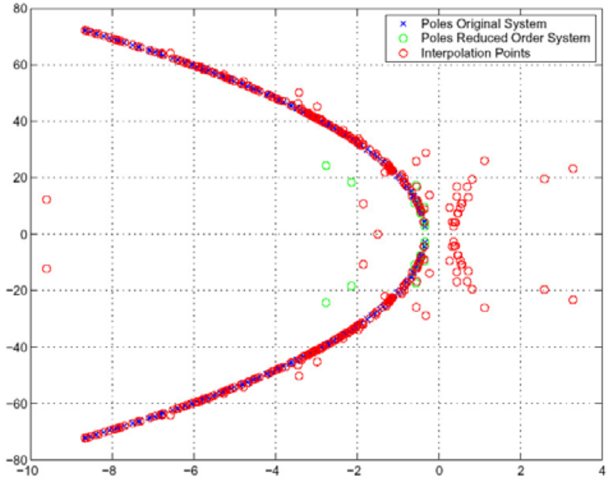
(— Original model, — BT, — SOBT)

Interpolation of large scale systems

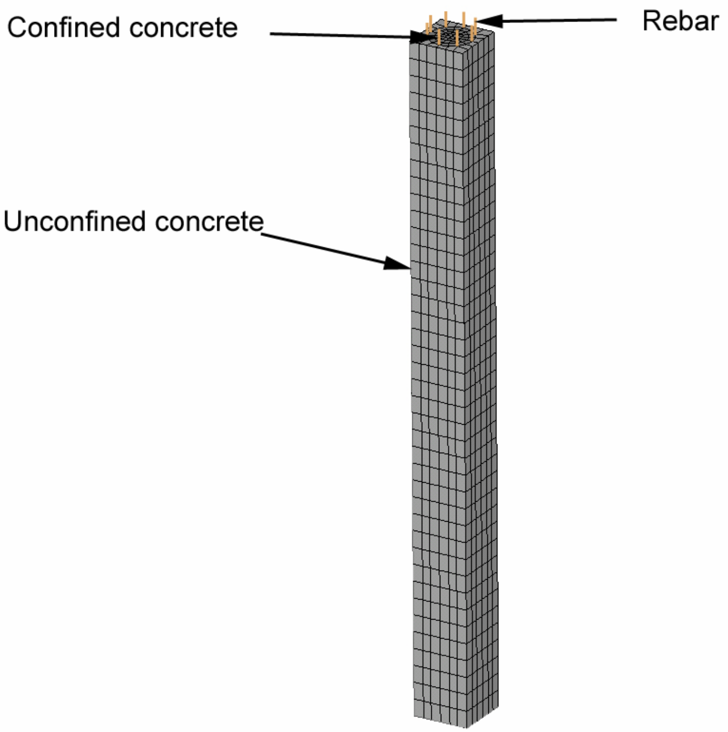
LA hospital building
has 2x26400 variables
but model is sparse



Eigenfrequencies closest
to the origin are typically
good interpolation points

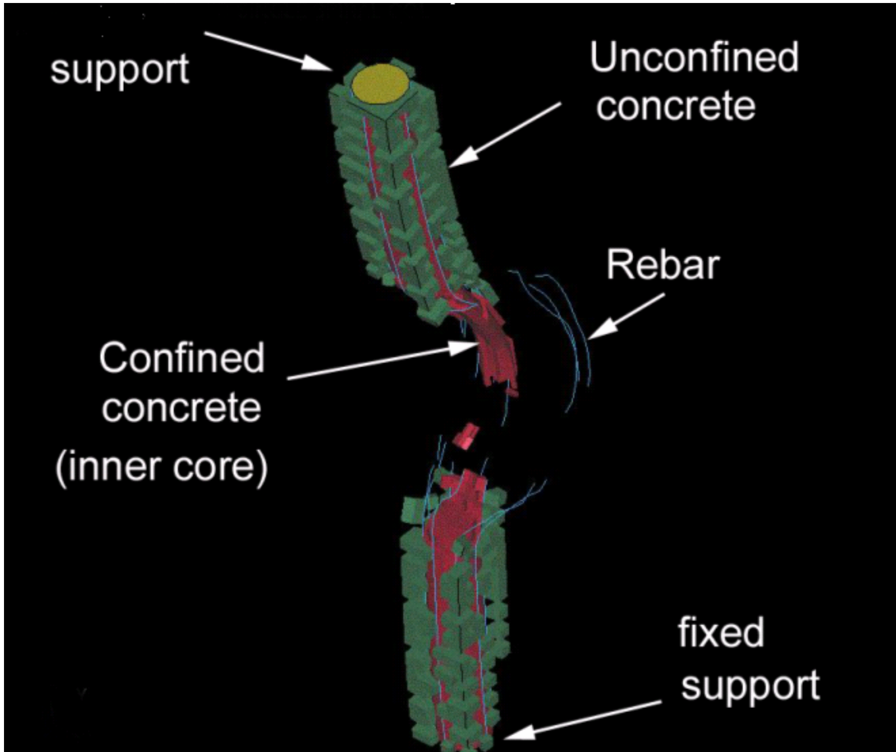


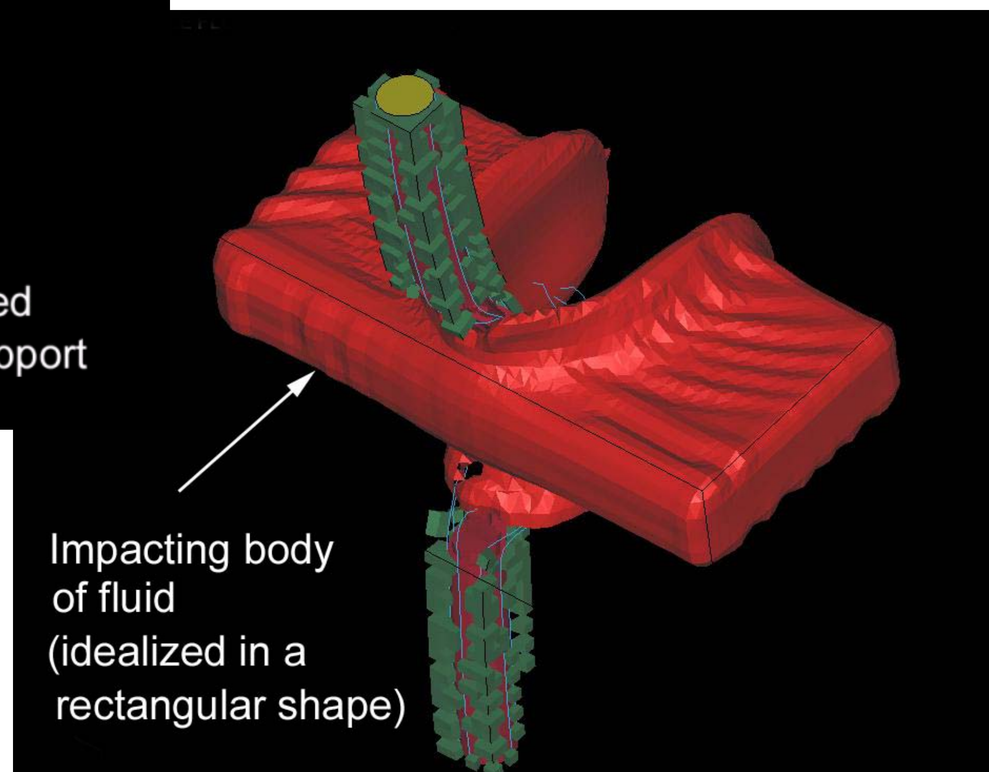
Structural simulation: case study



- Simulate the effects of crashing fluid into reinforced concrete
- Column model reproduces the behavior of spirally reinforced columns including different materials
- Fluid modeled by filling of elements in a (moving) grid
- IBM Regatta Power4 platform with 8 processors
- Model size: 1.2M elements, Run time: 20 hours

Structural simulation: case study

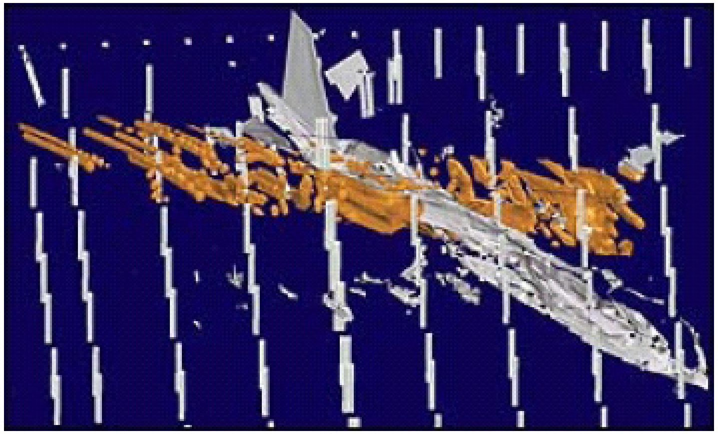




BBC report

of Sept. 2002

Computer clues to Pentagon attack



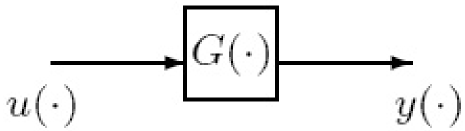
Orange shows fuel onboard as the plane crashed

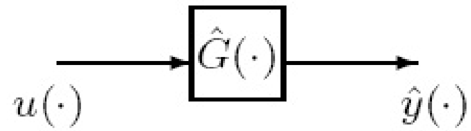
A computer simulation of the attack against the Pentagon last September could be used to design buildings that can withstand terrorist attacks.

The software used principles of physics to show how the plane's huge mass of fuel and cargo impacted the building.

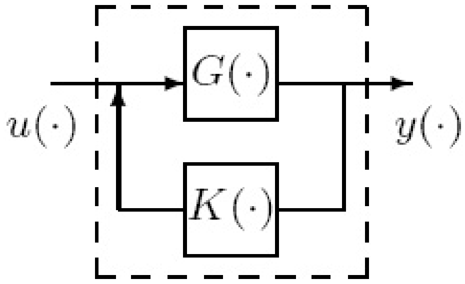
It could help design buildings such as hospitals and fire stations that would be more resistant to similar attacks.

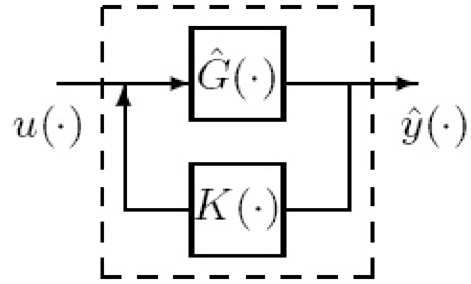
Interconnected systems





Open loop and closed loop approximations can be very different





The transfer function is now structured :

$$\min \|(I - G(s)K(s))^{-1}G(s) - (I - \hat{G}(s)K(s))^{-1}\hat{G}(s)\|_{H_\infty}$$

Several examples

Second order systems

$$\mathcal{S} \ : \ \begin{cases} (s^2M + sD + S)x(s) = Bu(s) \\ y(s) = Cx(s) \end{cases}$$

Cascaded systems

$$\mathcal{S} \ : \quad y(s) = [W_{out}G(s)W_{in}(s)]u(s)$$

Closed loop systems

$$\mathcal{S} \ : \quad \begin{bmatrix} y(s) \\ w(s) \end{bmatrix} = \begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix} \begin{bmatrix} u(s) \\ v(s) \end{bmatrix}$$

$$v(s) = C(s)w(s)$$

General interconnected systems

Interconnected systems

$$\begin{bmatrix} w_1(s) \\ \vdots \\ w_k(s) \end{bmatrix} = \begin{bmatrix} G_1(s) & & \\ & \ddots & \\ & & G_k(s) \end{bmatrix} \begin{bmatrix} v_1(s) \\ \vdots \\ v_k(s) \end{bmatrix}$$

Interconnect the components to each other and to the input and output $u(s), y(s)$

$$\begin{bmatrix} v_1(s) \\ \vdots \\ v_k(s) \end{bmatrix} = \begin{bmatrix} K_{11} & \dots & K_{1k} \\ \vdots & \ddots & \vdots \\ K_{k1} & \dots & K_{kk} \end{bmatrix} \begin{bmatrix} w_1(s) \\ \vdots \\ w_k(s) \end{bmatrix} + \begin{bmatrix} H_1 \\ \vdots \\ H_k \end{bmatrix} u(s)$$

$$y(s) = \begin{bmatrix} F_1 & \dots & F_k \end{bmatrix} \begin{bmatrix} w_1(s) \\ \vdots \\ w_k(s) \end{bmatrix}$$

Realize interconnected systems

Realize $G_i(s)$ of McMillan degree n_i as $C_i(sI_{n_i} - A_i)^{-1}B_i + D_i$ yielding :

$$A := diag\{A_i\}, \quad B := diag\{B_i\}, \quad C := diag\{C_i\}, \quad D := diag\{D_i\},$$

The interconnected system $T(s)$ is then realized by :

$$A_T := A + BK(I - DK)^{-1}C, \quad B_T := B(I - KD)^{-1}H,$$
$$C_T := F(I - DK)^{-1}C, \quad D_T := FD(I - KD)^{-1}H$$

It is not as bad as it looks since K is typically sparse, which is maintained in the realization of $T(s)$

Closed loop Gramians

Idea: For each subsystem $G_i(s)$, define $n_i \times n_i$ Gramians containing information about the energy distribution of the I/O map in x_i only.

Let us consider the controllability and observability Gramians of $T(s)$:

$$A_T P_T + P_T A_T^T + B_T B_T^T = 0 \quad , \quad A_T^T Q_T + Q_T A_T + C_T^T C_T = 0.$$

Decompose, with $P_{ij}, Q_{ij} \in \mathbb{C}^{n_i \times n_j}$

$$P_T = \begin{bmatrix} P_{11} & \dots & P_{1k} \\ \vdots & \ddots & \vdots \\ P_{k1} & \dots & P_{kk} \end{bmatrix} \quad , \quad Q_T = \begin{bmatrix} Q_{11} & \dots & Q_{1k} \\ \vdots & \ddots & \vdots \\ Q_{k1} & \dots & Q_{kk} \end{bmatrix},$$

Constrained Gramians

Energy interpretation of diagonal blocks $[P_{ii}]^{-1}$ and $[P^{-1}]_{ii}$

$$\min_{u,\bigcup_{j\neq i} x_j(0)} \|u(-\infty,0)\|^2_{\mathcal{L}_2} = x_i(0)^T [P_{ii}]^{-1} x_i(0)$$

$$\min_{u,\forall_{j\neq i} x_j(0)=0} \|u(-\infty,0)\|^2_{\mathcal{L}_2} = x_i(0)^T [P^{-1}]_{ii} x_i(0)$$

Similar for diagonal blocks $[Q_{ii}]^{-1}$ and $[Q^{-1}]_{ii}$

(see Enns, Liu, Varga, VV...)

Constrained Krylov spaces

If

$$(\hat{A}, \hat{B}, \hat{C}) = (Z^T A V, Z^T B, C V), \quad Z^T V = I$$

where

$$\mathcal{K}_k((\sigma I - A)^{-1}, (\sigma I - A)^{-1} B) \subseteq Im(V)$$

Then

$$(\hat{A}_T, \hat{B}_T, \hat{C}_T) = (Z^T A_T V, Z^T B_T, C_T V), \quad Z^T V = I$$

and

$$\mathcal{K}_k((\sigma I - A_T)^{-1}, (\sigma I - A_T)^{-1} B_T) \subseteq \mathcal{K}_k((\sigma I - A)^{-1}, (\sigma I - A)^{-1} B)$$

Open loop equals closed loop interpolation unless you change points !?

Conclusions

- Work has been initiated on several fronts
- Acquiring actual high-rise structural models (Purdue)
- Developing novel model reduction techniques and application on the above acquired full models (RICE, FSU, UCL)
- Development of sparse matrix parallel algorithms needed for model reduction and simulation (Purdue)
- Control via interconnected system (RICE, FSU, UCL)