Passive Macromodels of Microwave Subnetworks Characterized by Measured/Simulated Data

D. Saraswat, *Student member IEEE*, R. Achar, *Member IEEE*, M. Nakhla, *Fellow IEEE*
Department of Electronics, Carleton University, Ottawa, Ontario, Canada - K1S 5B6
Ph: (613) 520-5651, Fax: (613) 520-5708, Email: achar@doe.carleton.ca

Abstract - Passive macromodeling of microwave components, high-speed packages and interconnect modules characterized by simulated/measured data has generated immense interest during the recent years. This paper presents an efficient algorithm for addressing passivity of macromodels from simulated/measured data, based on linear formulation. Also one of the critical issues involved in such macromodels is the passivity check and compensation. For this purpose, a new theorem and an efficient algorithm is presented. Examples are presented to demonstrate the validity and efficiency of the proposed algorithm.

I. INTRODUCTION

Recently, characterization and simulation of linear subnetworks based on measured/simulated (*tabulated*) data (obtained either directly from measurements or from rigorous full-wave electromagnetic simulation) has become a topic of intense research. Important applications of such a characterization include microwave devices, high-speed packages, vias, nonuniform transmission lines and on-chip passive components, such as inductors and transformers [1]-[6]. However, transient simulation of such frequency-dependent tabulated data in the presence of nonlinear devices to obtain a global electrical assessment is a CPU expensive process due to the mixed frequency/time problem. Prominent approaches to solve this difficulty [1]-[10] are based on approximating the tabulated data through rational-functions and subsequently synthesizing a SPICE compatible macromodel/netlist from such an approximation. However, the primary challenge in such approaches is ensuring the passivity of the maromodel. Passivity is an important property, because stable but non-passive models may lead to unstable systems when connected to other passive components.

Conventional approaches in the literature on imposing passivity constraints lead to nonlinear optimization formulation, which can be CPU expensive [2]. Alternative approaches use constraints such as, *every first or second order pole-residue pair must strictly conform to passivity relations*, which is sufficient but not necessary, [3]. It is to be noted that, most practical circuits do not obey these conditions and strict enforcement of these conditions may lead to convergence problems, inaccurate and CPU expensive macromodels. Techniques such as the one in [5] formulate the macromodel synthesis problem as linear unconstrained problem.

In these algorithms [2]-[5], macromodels are checked for any passivity violation and compensated if necessary. Hence the success of these algorithms depends on the *quality of the macromodel prior to post-processing, that is, the extent of their conforming to passivity conditions*. Post-processing can be quite effective if the passivity violation is very minor; on the other hand, if the violation is significant, it can lead to inaccurate macromodels. Another critical issue involved here is the passivity check. Traditional approach for this purpose is based on *frequency-sweep of eigenvalues of the real-part of the admittance matrix (Re(Y(s)))* of the macromodel. However, this approach suffers from several drawbacks, such as up to what frequency to sweep, how fine the sweep should be and how to identify the exact locations of violation.

This paper describes an algorithm for passive macromodeling of microwave subnetworks characterized by tabulated data, with the following new contributions:

1) A new set of linear passivity conforming constraints are presented to ensure macromodel passivity. Since the constraints are linear, macromodel generation is highly CPU efficient as compared to using traditional nonlinear constraints.
2) A new theorem is presented which enables systematic passivity check and compensation. It enables: (a) *performing passivity check without requiring any frequency sweep of eigenvalues of Re(Y(s))*, (b) *indentifying exact locations of any negative eigenvalues*, (c) *identifying any negative eigenvalues of Re(Y(s)) independent of where they are occurring in the frequency spectrum*.

The new theorem is based on formulation of the Hamiltonian matrix of the state-space equations representing the macromodel. The knowledge of the exact locations of negative eigenvalues of *Re(Y(s))* is very crucial as it greatly helps the passivity-compensation process. Numerical examples are presented to validate the efficiency and accuracy of the proposed algorithm.

II. PROBLEM FORMULATION

The tabulated data can be multi-port scattering (***S***), admittance (***Y***), impedance (***Z***), transmission (***T***) or hybrid (***H***) parameters. For the ease of presentation, in this paper it is assumed that the *Y*-parameter data is given. The admittance matrix of a *m*-port subnetwork can be written in terms of a rational-approximation as

0-7803-7695-1/03/$17.00 © 2003 IEEE

$$Y(s) = \left[Y_{ij}(s)\right]; \quad Y_{ij}(s) = \frac{(a_0^{(i,j)} + a_1^{(i,j)}s + \ldots + a_L^{(i,j)}s^L)}{(b_0^{(i,j)} + b_1^{(i,j)}s + \ldots + b_N^{(i,j)}s^N)}$$
$$(i, j \in 1 \ldots m) \tag{1}$$

The challenge here is to ensure both the accuracy and passivity of the multiport macromodel. The loss of macromodel passivity can be a serious problem because transient simulations may encounter artificial oscillations. A network with admittance matrix $\boldsymbol{Y}(s)$ is passive [5], iff,

(a) $\boldsymbol{Y}(s^*) = \boldsymbol{Y}^*(s)$, where '$*$' is the complex conjugate operator.

(b) $\boldsymbol{Y}(s)$ is a positive real (PR) matrix, i.e., the product $\boldsymbol{z}^{*t}[\boldsymbol{Y}^t(s^*) + \boldsymbol{Y}(s)]\boldsymbol{z} \geq 0$ for all complex values of s with $Re(s) > 0$ and any arbitrary vector $\boldsymbol{z}$.

Condition (a) is automatically satisfied since the complex poles/residues of the transfer function are always considered along with their conjugates, leading to only real coefficients in rational functions of $\boldsymbol{Y}(s)$. However, ensuring condition (b) is not easy. For the practical case of networks with symmetric admittance matrices, condition (b) implies that:

$$Real(\boldsymbol{Y}(s)) = [\boldsymbol{Y}^t(s^*) + \boldsymbol{Y}(s)]/2 = \boldsymbol{F}(s) \tag{2}$$

must be positive definite for all s with $Re(s) > 0$.

III. Development of the Proposed Algorithm

In order to ensure both accuracy and passivity of the macromodel a new algorithm is developed. The first step involves computation of multiport pole-set. The second step computes the residues, subject to certain linear constraints which help to ensure macromodel passivity. The third step checks for any possible passivity violation and corrects in case of violation. A brief discussion of these steps is given below.

Step1: Computation of Multiport Dominant Pole-Set

This step uses the pole identification algorithm of the vector fitting approach [7] to obtain an accurate set of poles. Here, an initial guess of poles is considered and a scaling function is introduced. With this initial guess of poles, the scaled function is accurately fitted, from which an accurate set of poles are computed. The algorithm ensures that the multiport admittance matrix is evaluated based on common pole set.

Step 2: Formulation of Residue Equations and Passivity Conforming Linear Constraints

In this step, residues are computed such that the macromodel satisfies the passivity conditions outlined in section II. Straight-forward application of passivity constraints can lead to nonlinear optimization problem. In order to overcome this difficulty, the following linear constraints, which help to retain the passivity of the macromodel are proposed. Let w_{max} be the frequency corresponding to the highest given data point. Let the common pole set ($\boldsymbol{P}$) in the ascending order be denoted as

$$\boldsymbol{P} = \left[p_1, p_2 \; \cdots \; p_{max0}, p_{max1} \cdots, p_{max}\right];$$
$$(\mathrm{imag}(p_1) < \ldots < \ldots < \mathrm{imag}(p_{max}))$$
$$(\mathrm{imag}(p_{max0}) < w_{max} < \mathrm{imag}(p_{max1})) \tag{3}$$

Next, each Y_{ij} can be expressed using the pole-residue relation and the frequency response as:

$$c^{i,j} + \frac{k_1^{i,j}}{s_h - p_1} + \frac{k_2^{i,j}}{s_h - p_2} + \ldots + \frac{k_q^{i,j}}{s_h - p_q} = \psi^{i,j}(s_h);$$

$q \rightarrow$ total number of poles; $p_l, k_l \rightarrow l^{th}$ pole-residue pair;

$\psi^{i,j}(s_h) \rightarrow$ given tabulated data at h^{th} frequency point s_h (4)

Equating both the real and imaginary parts of (4) separately at all the data points, we can write

$$\begin{bmatrix} \boldsymbol{G}_r \\ \boldsymbol{G}_e \end{bmatrix} \begin{bmatrix} \boldsymbol{K}_r^{i,j} \\ \boldsymbol{K}_e^{i,j} \end{bmatrix} = \begin{bmatrix} \psi_r^{i,j} \\ \psi_e^{i,j} \end{bmatrix}; \qquad or \qquad \boldsymbol{G}\boldsymbol{K}^{i,j} = \psi^{i,j} \tag{5}$$

where the subscripts r and e correspond to the *real* and *imaginary* parts, respectively, for the corresponding parameters/formulations (vector $\boldsymbol{K}_r^{i,j}$ also includes the direct coupling constant $c^{i,j}$). Equation (5) is solved subject to the following new passivity conforming constraints:

$$\left.\begin{array}{ll} \boldsymbol{G}^t\boldsymbol{G}\boldsymbol{K}^{i,j} = \boldsymbol{G}^t\psi^{i,j} & c^{i,j} \geq 0; \quad \text{(a)} \\ \text{such that,} & \boldsymbol{G}_r\boldsymbol{K}^{i,j} \geq \psi_r^{i,j}; \quad \text{(b)} \end{array}\right\} for \;\; (i = j)$$
$$\left.\begin{array}{l} c^{i,j} = 0; \quad \text{(c)} \\ \boldsymbol{P} = \left[p_1, \;\; p_2, \;\; \ldots \, p_{max0}\right]; \quad \text{(d)} \end{array}\right\} for \;\; (i \neq j) \tag{6}$$

Since the computed model matches the tabulated data (it is assumed that the original data conforms to passivity conditions) accurately up to p_{max0}, we have $\boldsymbol{z}^{*t}[Re(\boldsymbol{Y}(s))]\boldsymbol{z} \geq 0$ for $Re(s) > 0$, in the region $(0 \leq w \leq p_{max0})$. Condition-(6a) is necessary to ensure that $\boldsymbol{z}^{*t}[Re(\boldsymbol{Y}(s))]\boldsymbol{z} \geq 0$ at $s = \infty$ for $Re(s) > 0$. Constraint-(6b) will guarantee that the real part of driving point admittances remains greater than zero in the region $(0 \leq w \leq w_{max})$. Conditions-6(c,d) help to ensure $\boldsymbol{z}^{*t}[Re(\boldsymbol{Y}(s))]\boldsymbol{z} \geq 0$ for $Re(s) > 0$ in the region $(p_{max0} \leq w \leq \infty)$.

Enforcing the above conditions and performing linear constrained optimization, will lead to passive macromodels for most cases of practical measured/simulated data. It is important to note that, for macromodels thus generated, the post-processing or compensation requirement is very minimum. Also, since the constraints are linear, macromodel generation is highly CPU efficient. However, it is to be noted that, since the above constraints are not strict passivity enforcing conditions, there may be minor chances of passivity violation, which may require post compensation. The details of passivity check and compensation (third step) are given in the next section.

IV. Passivity Check and Compensation

New results are presented in this section which enable systematic passive check and compensation. The main features of these theorems are that, *a) passivity check can be performed without requiring any frequency sweep of eigenvalues of Re(Y(s)), b) can identify exact locations of any negative eigenvalues Re(Y(s)), independent of where they are occurring in the frequency spectrum.* A brief discussion of the new approach is given below. Using the *m*-port pole-residue model (1), a state-space system with minimum realization [10] can be obtained as

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \qquad A \in \Re^{n\times n} \quad C \in \Re^{m\times n} \\ y(t) &= Cx(t) + Du(t) \qquad B \in \Re^{n\times m} \quad D \in \Re^{m\times m} \end{aligned} \tag{7}$$

Theorem 1: The minimum realized system $\langle A, B, C, D\rangle$ is passive iff the following Hamiltonian Matrix (M) [11] has no imaginary eigenvalues,

$$M = \begin{bmatrix} A - B(D+D^t)^{-1}C & B(D+D^t)^{-1}B^t \\ -C^t(D+D^t)^{-1}C & -A^t + C^t(D+D^t)^{-1}B^t \end{bmatrix} \tag{8}$$

If it is found that the matrix M in (8) has any imaginary eigenvalue, then the system is not passive. In this context, a new theorem is introduced, which helps to identify the exact locations at which *real part of the transfer-function (admittance) matrix ($F(s)$* - defined in (2)) becomes singular (i.e. where its eigenvalues become zero).

Theorem 2: $F(j\omega_0)$ is singular, iff $j\omega_0$ is an imaginary eigenvalue of M, provided $D + D^t > 0$.

The detailed proof of above theorems is not given due to the lack of space. In brief, using (2), (7) and (8), and after certain manipulations, it can be shown that

$$\begin{aligned} &det(D+D^t)det(j\omega I - M) \\ &\quad = det(j\omega I - A)det(j\omega I + A^t)det(F(j\omega)) \end{aligned} \tag{9}$$

From (9) it is evident (under assumption that A has no imaginary eigenvalue) that, if $j\omega_0$ is an eigenvalue of M, then $F(j\omega)$ is singular at $j\omega_0$. Which implies that an imaginary eigenvalue of the Hamiltonian matrix M corresponds to the frequency at which $F(j\omega)$ is singular. Two successive imaginary eigenvalues (while arranged in ascending order) of M define the frequency interval for which one of the eigenvalues of $F(j\omega)$ remains negative.

The information of the exact locations where an eigenvalue of $F(j\omega)$ becomes zero (and start reversing its sign) is very crucial as its knowledge will greatly help the passivity-compensation process. If a negative eigenvalue spectrum exists, it could be easily corrected by inserting additional artificial poles [5] and using the above information. Alternatively, the above information can be used to accurately define linear constraints for passivity compensation [8], details of which is not given here due to the lack of space. Having ensured the passivity of the macromodel, it can be easily linked to nonlinear simulators for performing transient analysis.

V. Computational Results

The proposed algorithm was performed on measured **Y**-parameters (data is given up to 6GHz) of a 3-port distributed subnetwork [5]. Fig. 1 shows the accuracy comparison of the macromodel magnitude responses with the original data, and they match accurately. If we use the constraint 6(b), the macromodel didn't require any passivity compensation. However, to illustrate the proposed passivity check algorithm of section III, we carried out the fitting process without using constraint 6(b), in which case the generated macromodel became non-passive. This was tested using Theorem-1, and the complete eigenvalue distribution of the corresponding Hamiltonian matrix is given in Fig. 2a (in which six pair of complex eigenvalues were found to be purely imaginary). For the purpose of clarity, Fig. 2b shows an enlarged view of the eigenvalue spread near the imaginary axis and also shows the exact numerical values of the imaginary eigenvalues.

According to Theorem 2 the above imaginary eigenvalues correspond to the location where the *Re(**Y**(s))* becomes singular. Fig. 3 confirms this result, which shows the eigenvalue spectrum of *Re(**Y**(s))*. As seen, *Re(**Y**(s))* becomes singular at six frequency points (corresponding to the imaginary eigenvalues of the Hamiltonian matrix). Next, using the above information, passivity compensation was performed with the method in [8] (Fig. 4). Fig. 5 shows the comparison of macromodel transient response (in the presence of nonlinear terminations) with the SPICE simulation of the original circuit, which match accurately.

VI. Conclusions

In this paper an algorithm is presented for efficient passive macromodeling of subnetworks characterized by tabulated data. Also an efficient algorithm is presented for passivity check and identifying the locations where real-part of admittance matrix becomes singular. This information is crucial for efficient post passivity compensation.

VII. References

[1] W. T. Beyene, and J. E. Schutt-Aine, "Accurate frequency-domain modelling and efficient simulation of high-speed packaging interconnects", *IEEE Transactions MTT*, pp. 1941-1947, Oct. 1997.

[2] S. Min and M. Swaminathan, "Efficient Construction of Two Port Passive Macromodels", *10th Topical Meeting on EPEP, Cambridge*, pp. 230-232, October 2001.

[3] J. Morsey and A. C. Cangellaris, "Passive Realization of Interconnect Models from Measured Data", *10th Topical Meeting on EPEP*, Cambridge, pp. 47 - 50, Oct. 2001.

[4] C. P. Coelho, J. R. Phillips, and L. M. Silveira, "Robust Rational Function Approximation Algorithm for Model Generation", *Proc. 36th DAC*, pp. 207-212, June 1999.

[5] R. Achar, P. Gunupudi, M. Nakhla and E. Chiprout, "Passive Interconnect Reduction Algorithm for Distributed/Measured Networks", *TCAS-II*, pp. 287-301, April 2000.

[6] D. Saraswat, R. Achar and M. Nakhla, "A Fast Algorithm and Practical Considerations For Passive Macromodeling Measured/Simulated Data ", *IEEE 11th Topical Meeting on EPEP, Monterery, CA*, pp. 297-300, October 2002.

[7] B. Gustavsen and A. Semlyen, "Rational Approximation of Frequency Domain Responses By Vector Fitting", *IEEE Trans on Power Delivery*, pp. 1052-1061, July 1999.

[8] B. Gustavsen and A. Semlyen, "Enforcing Passivity for Admittance Matrices Approximated by Rational Functions", *IEEE Trans. on Power Systems*, pp. 97-104, February 2001.

[9] R. Neumayer, F. Haslinger, A. Stelzer and R. Weigel, "On the Synthesis of Equivalent Circuit Models for Multiports Characterized by Frequency-Dependent Parameters", *IMS-2002 Digest*, Seattle, June 2002.

[10] R. Achar and M. Nakhla, *Minimum Realization of High-Speed Interconnect Macromodels*, Chapter III: Signal Propagation on Interconnects, pp. 23-45, Kluwer Pub., 1998.

[11] Pablo A. Parrilo,"*Structured Semidefinite Programs and Semialgebraic Geometry Methods in Robustness and Optimization*", PhD. thesis, California Inst. of Tech., Pasadena.

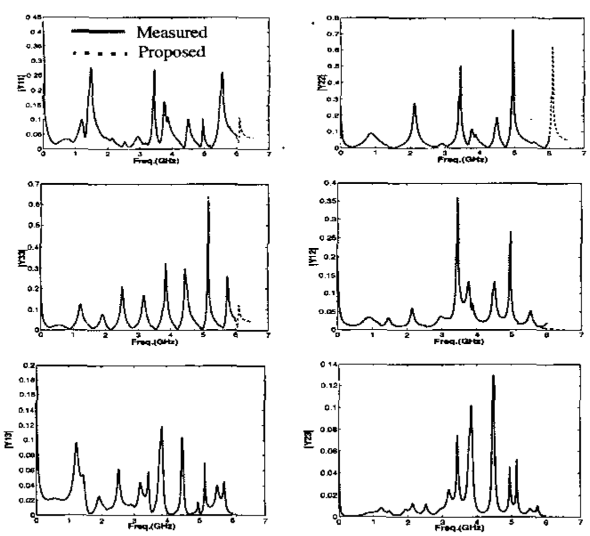

Fig. 1. Three-port admittance parameters

Fig. 2.(a), (b): Eigenvalue Spectrum of Hamiltonian Matrix

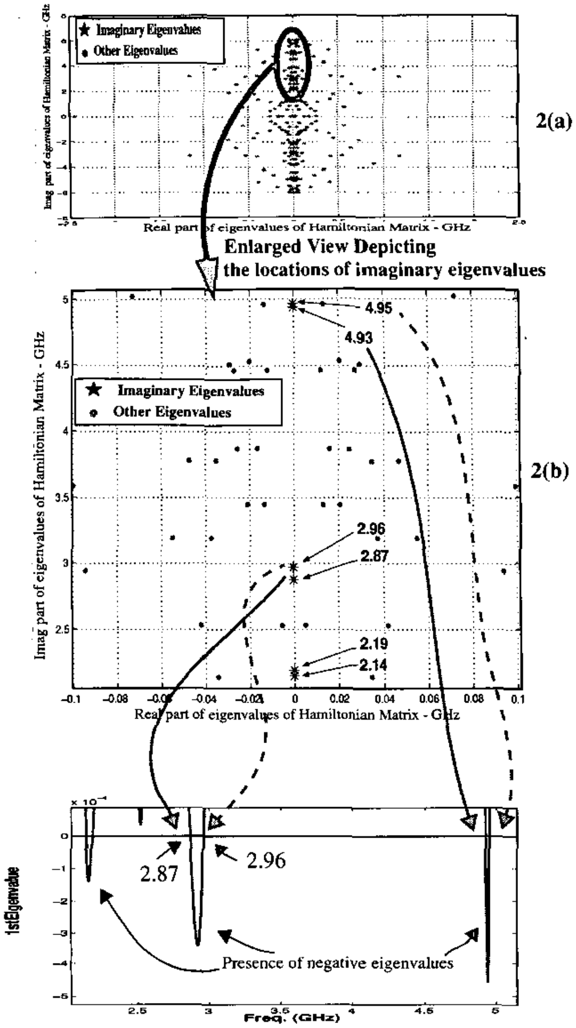

Fig. 3. Eigenvalue v/s freq. of *Re(Y(s))* - with passivity violation

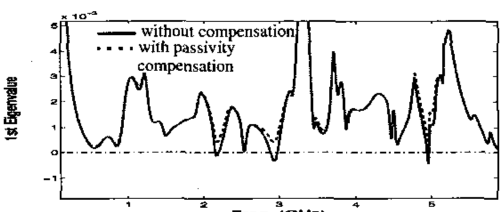

Fig. 4. Eigenvalue v/s freq. of Re(Y(s)) - with compensation

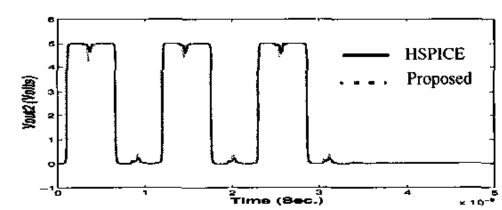

Fig. 5. Macromodel Transient Responses