ON PASSIVITY ENFORCEMENT FOR MACROMODELS OF S-PARAMETER BASED TABULATED SUBNETWORKS

D. Saraswat, R. Achar and M. Nakhla
Department of Electronics, Carleton University, Ottawa, Ontario, K1S 5B6
Tel (613) 520-5651; Fax: (613) 520-5708; Email: achar@doe.carleton.ca

Abstract— **Passive macromodeling of EM subnetworks and high-speed modules characterized by S-parameters has generated immense interest during the recent years. This paper presents a comparison of two techniques for the passivity compensation of macromodels of S-parameter based subnetworks. The formulation and implementation of the two techniques are discussed. Numerical examples are presented to validate the theory presented in this paper.**

I. INTRODUCTION

Due to the continually increasing operating frequencies and circuit densities/complexities, characterization and simulation of high-speed electromagnetic subnetworks based on S-parameters has become a topic of intense research. The S-parameters can be obtained either directly from measurements or from rigorous full-wave electromagnetic simulation. Important applications of such a characterization include microwave devices, high-speed packages, vias, nonuniform transmission lines, antennas and on-chip passive components, such as inductors and transformers. However, for the transient simulation of such frequency-dependent tabulated data, a simulatable model is required. All the prominent techniques for macromodel generation approximate the tabulated data with a rational approximation [1]-[6]. Although most of these techniques lead to stable approximations (having poles with negative real parts), they may not guarantee the passivity of the macromodel. However, passivity is an important property, because stable but non-passive models may lead to unstable systems when connected to other passive components.

Recently, several techniques for passivity enforcement of macromodel have been proposed in the literature. Some of these techniques may be too constraining, while others based on the convex optimization may be limited to small problem size due to large computational cost [4]. This paper describes two techniques for passivity compensation, based on the perturbation of the rational approximation of the S-parameter tabulated data [5], [6]. The first technique uses first-order perturbation formulation to enforce passivity (we will call this technique "passivity compensation by first-order perturbation", in abbreviated form as PCFOP). In order that the formulation satisfies the first-order perturbation requirement, compensation is done in small increments at a time. The second technique uses the linear matrix inequality (LMI) formulation, for carrying out the compensation (we will call this technique "passivity compensation by linear matrix inequality", in abbreviated form as PCLMI). A comparative study of the above two approaches is presented. Numerical examples are given to validate the described algorithms.

II. PROBLEM FORMULATION

The scattering parameters ($\boldsymbol{S}$) are widely used to characterize multiport subnetworks at higher frequencies, by relating the incident travelling waves ($\boldsymbol{a}$) and the reflected travelling waves ($\boldsymbol{b}$) as

$$\boldsymbol{b} = \boldsymbol{S}\boldsymbol{a} \tag{1}$$

The rational approximation of S-parameters of a m-port subnetwork can be written as

$$\boldsymbol{S}(s) = \left[S_{ij}(s)\right]; \qquad S_{ij}(s) = \frac{(a_0^{(i,j)} + \ldots + a_L^{(i,j)} s^L)}{(b_0^{(i,j)} + \ldots + b_N^{(i,j)} s^N)} \qquad (i,j \in 1 \ldots m) \tag{2}$$

Several algorithms can be found which can compute the rational approximation for the given tabulated data [2]-[6], [8]. However, the challenge here is to ensure both the accuracy and passivity of the multiport macromodel. The loss of macromodel passivity can be a serious problem because transient simulations may encounter artificial oscillations. A network with scattering matrix $\boldsymbol{S}(s)$ is passive [7] iff,

(a) $\boldsymbol{S}(s^*) = \boldsymbol{S}^*(s)$, '*' is the complex conjugate operator.

(b) $\boldsymbol{S}(s)$ is bounded real, i.e., $\|\boldsymbol{S}(j\omega)\| \le 1$ or $\boldsymbol{I} - (\boldsymbol{S}(j\omega))^H \boldsymbol{S}(j\omega) \ge \boldsymbol{0}$ for $\omega \in \Re$.

Condition (a) is automatically satisfied in rational-function based approximations [8], since the complex poles/residues of the transfer function are always considered along with their conjugates, leading to only real coefficients in rational functions of $\boldsymbol{S}(s)$. However, ensuring condition (b) is not easy (straight-forward formulation can lead to computationally expensive nonlinear optimization, which can also suffer from non-convergence).

In order to address the above problem, two techniques for passivity compensation, based on the perturbation of the rational approximation of the S-parameter tabulated data [5], [6] are discussed in this paper. Both techniques start from the pole-residue approximation (2) using methods described in [5], [6]. This approximation is converted into state-space representation and checked for passivity based on method described in [5]. A comparative study of the above two approaches is presented.

In the following section, we review the procedure for systematic passivity checking and identification of local bandwidths of passivity violation. The details of the procedure may be found in [5].

III. PASSIVITY CHECK AND DETERMINATION OF REGIONS OF LOCAL PASSIVITY VIOLATION

Consider the m-port pole-residue macromodel represented by (2). Corresponding state-space representation Λ can be obtained as

$$\left.\begin{aligned}\dot{\boldsymbol{x}}(t) &= \boldsymbol{A}\boldsymbol{x}(t)+\boldsymbol{B}\boldsymbol{u}(t)\\ \boldsymbol{y}(t) &= \boldsymbol{C}\boldsymbol{x}(t)+\boldsymbol{D}\boldsymbol{u}(t)\end{aligned}\right\} \leftrightarrow \Lambda \tag{3}$$

where $\boldsymbol{A} \in \Re^{n\times n}$, $\boldsymbol{B} \in \Re^{n\times m}$, $\boldsymbol{C} \in \Re^{m\times n}$, and $\boldsymbol{D} \in \Re^{m\times m}$. The relationship between the input $\boldsymbol{u}(t)$ and output $\boldsymbol{y}(t)$ can be obtained as

$$\boldsymbol{S}(j\omega) = \boldsymbol{C}(j\omega\boldsymbol{I}-\boldsymbol{A})^{-1}\boldsymbol{B}+\boldsymbol{D} \tag{4}$$

It is assumed that the matrix $\boldsymbol{A}$ has no imaginary eigenvalues (only stable poles constitute the macromodel) and the matrix $\boldsymbol{D}$ (constituted by direct coupling constants) has norm less than one. The pole-residue approximation (2) of tabulated data is not guaranteed to satisfy the passivity conditions. Hence, the corresponding state-space system (3) is checked for passivity based on the following theorems:

Theorem 1: The state-space system $(\boldsymbol{A},\boldsymbol{B},\boldsymbol{C},\boldsymbol{D})$ is bounded real (passive) iff the following Hamiltonian Matrix ($\boldsymbol{M}$) [7] has no imaginary eigenvalues,

$$\boldsymbol{M} = \begin{bmatrix} \boldsymbol{A}+\boldsymbol{B}(\boldsymbol{I}-\boldsymbol{D}^t\boldsymbol{D})^{-1}\boldsymbol{D}^t\boldsymbol{C} & \boldsymbol{B}(\boldsymbol{I}-\boldsymbol{D}^t\boldsymbol{D})^{-1}\boldsymbol{B}^t \\ -\boldsymbol{C}^t(\boldsymbol{I}-\boldsymbol{D}\boldsymbol{D}^t)^{-1}\boldsymbol{C} & -\boldsymbol{A}^t-\boldsymbol{C}^t\boldsymbol{D}(\boldsymbol{I}-\boldsymbol{D}^t\boldsymbol{D})^{-1}\boldsymbol{B}^t \end{bmatrix} \tag{5}$$

If no imaginary eigenvalues are found, it automatically implies that the macromodel is passive. If there are imaginary eigenvalues found, the following theorem helps in identifying the exact locations of passivity violation.

Theorem 2: $\boldsymbol{S}(j\omega_0)$ has a maximum singular value equal to one (i.e. the norm equal to one) iff $j\omega_0$ is an imaginary eigenvalue of $\boldsymbol{M}$, provided $\boldsymbol{A}$ has no imaginary eigenvalues and $\boldsymbol{D}$ does not have a singular value equal to one [9].

Theorem 2 gives a very useful information about the exact frequency points where the macromodel transits from being passive to non-passive. But, this information alone is not enough to carry out passivity compensation. In addition, we need the exact frequency bandwidths of passivity violation as well as the location of maximum passivity violation in each bandwidth of violation. A systematic method for this, based on the frequency points obtained from Theorem 2 was presented in [5]. Using this information, in the next section, we describe two techniques for passivity compensation.

IV. PASSIVITY COMPENSATION

Using the information from section III, we describe two techniques for obtaining passive approximation Λ_p for a given non-passive model Λ, such that the induced perturbation in the input-output responses is minimized. To be more precise, we keep the state-space matrices $\boldsymbol{A},\boldsymbol{B},\boldsymbol{D}$ fixed and perturb the matrix $\boldsymbol{C}$ containing residues by an amount $\Delta\boldsymbol{C}$. In such a case if we denote the induced perturbation in $\boldsymbol{S}(j\omega)$ by $\Delta\boldsymbol{S}(j\omega)$, it can be verified that [10]

$$\begin{aligned}\|\Delta\boldsymbol{S}\|_2^2 &= \int_{-\infty}^{\infty}\|\Delta\boldsymbol{S}(j\omega)\|_F^2 d\omega = \int_{-\infty}^{\infty}\sum_{i=1}^{m}\sum_{j=1}^{m}\left|\Delta S_{ij}(\omega)\right|^2 d\omega \\ &= \int_{-\infty}^{\infty}\sum_{i=1}^{m}\sum_{j=1}^{m}\left|\Delta S_{ij}(t)\right|^2 dt = trace(\Delta\boldsymbol{C}\ \boldsymbol{P}\ \Delta\boldsymbol{C}^t)\end{aligned} \tag{6}$$

where $\|\Delta\boldsymbol{S}(j\omega)\|_F$ is the Frobenius norm of $\Delta\boldsymbol{S}(j\omega)$, m is the number of ports and $\boldsymbol{P}$ is the controllability Grammian obtained by solving the following Lyapunov equation

$$\boldsymbol{A}\boldsymbol{P}+\boldsymbol{P}\boldsymbol{A}^t+\boldsymbol{B}\boldsymbol{B}^t = 0 \tag{7}$$

IV.1 Compensation by first-order perturbation (PCFOP)

With a small perturbation $\Delta\boldsymbol{C}$ in matrix $\boldsymbol{C}$, the perturbed scattering model represented by $\hat{\boldsymbol{S}}(j\omega)$ can be expressed as

$$\hat{S}(j\omega) = (C+\Delta C)(j\omega I - A)^{-1}B + D = S(j\omega) + \Delta S(j\omega) \quad (8)$$

where

$$\Delta S(j\omega) = \Delta C(j\omega I - A)^{-1}B \quad (9)$$

If the model (4) is nonpassive (i.e. $I-(S(j\omega))^H S(j\omega)$ is negative definite), matrix C is perturbed by ΔC such that the perturbed model (8) satisfies Condition (b).

The perturbation ΔC is calculated by using the first order eigenvalue perturbation formula [11]. The perturbed model $\hat{S}(j\omega) = S(j\omega) + \Delta S(j\omega)$ should be such that it satisfies the following condition at the frequency points of violation

$$I - (\hat{S}(j\omega))^H \hat{S}(j\omega) \geq 0 \quad (10)$$

or (for simplicity dropping $j\omega$)

$$I - S^H S - S^H \Delta S - \Delta S^H S - \Delta S^H \Delta S \geq 0 \quad (11)$$

Neglecting the second order term in the perturbation ΔS, (11) implies that if the original model is nonpassive, i.e. an eigenvalue of $I - S^H S$ is negative by an amount $\Delta\lambda$, we perturb $I - S^H S$ by $-S^H \Delta S - \Delta S^H S$ such that (11) is satisfied. For the first-order perturbation condition be satisfied, the perturbation in C is carried out in increments. This results in the following system of equations [11]

$$\Delta\lambda_i = \frac{v^t(-S^H \Delta S_i - \Delta S_i^H S)u}{v^t u} \quad (12)$$

where v and u are the left and right eigenvectors of $I - S^H S$ respectively, $\Delta\lambda_i$ is the amount of correction in eigenvalue of L.H.S of (11) under which the first-order formulae is valid. Next, (12) can be formulated as a least-square problem

$$WX = \Delta\lambda_i \quad (13)$$

where X is the vector of unknowns of the perturbation matrix ΔC, while W is composed of entries of matrix A, B, C, D and ω. The problem in (13) is solved iteratively at the frequency points of maximum violation in each region of the passivity violation, with the constraint that the error in the response (6) is minimized.

IV.2 Compensation by linear matrix inequality (PCLMI)

In this section we describe the technique based on the Linear Matrix Inequality formulation of (11). Neglecting the second order term in the perturbation ΔS, (11) can be written as the LMI

$$I - S^H S - S^H \Delta S - \Delta S^H S \geq 0 \quad (14)$$

where, the matrix on the L.H.S. of (14) is complex-valued and Hermitian.

The LMI solvers are written for real-valued matrices and cannot directly handle LMI problems involving complex valued matrices. However, complex-valued LMIs can be turned into real-valued LMIs by observing that a complex Hermitian matrix $H(x)$ satisfies

$$H(x) > 0 \quad (15)$$

if and only if

$$\begin{bmatrix} Re(H(x)) & Im(H(x)) \\ -Im(H(x)) & Re(H(x)) \end{bmatrix} > 0 \quad (16)$$

Using this information, (14) can be written in the form of inequality suitable for Linear Matrix Inequality (LMI) solvers. In our implementation, the above LMI is solved subjected to the constraint (6) in the MATLAB LMI solver. The difference in this approach is that the desired perturbation ΔC is calculated in one go at one frequency point of violation, as against the technique presented in Section IV.1, where correction is done at a frequency point in several small steps subjected to satisfying the first-order perturbation formulae.

V. Computational Results

For the comparison we considered RJ-45 connector consisting of eight-ports. The frequency dependent S-parameters tabulated data was approximated by the algorithm described in [5]. The resulting size of the state-space system was 80×80. The macromodel was checked for passivity and it was found to be non-passive, illustrated by the plot of norm of $S(j\omega)$ (in solid line) of the macromodel in Fig. 2. This non-passive model was compensated by using the two techniques described in section IV. The resulting responses after compensation are shown in Fig. 1 while the norm of $S(j\omega)$ is plotted in Fig. 2. Both the techniques took approximately 8 minutes for compensation. Frequency responses from both the techniques had a comparable accuracy.

VI. Conclusions

In this paper, two techniques and their comparison for passivity compensation, based on the perturbation of the rational approximation of the S-parameter tabulated data are described. Necessary formulations and numerical examples are presented for validation purposes.

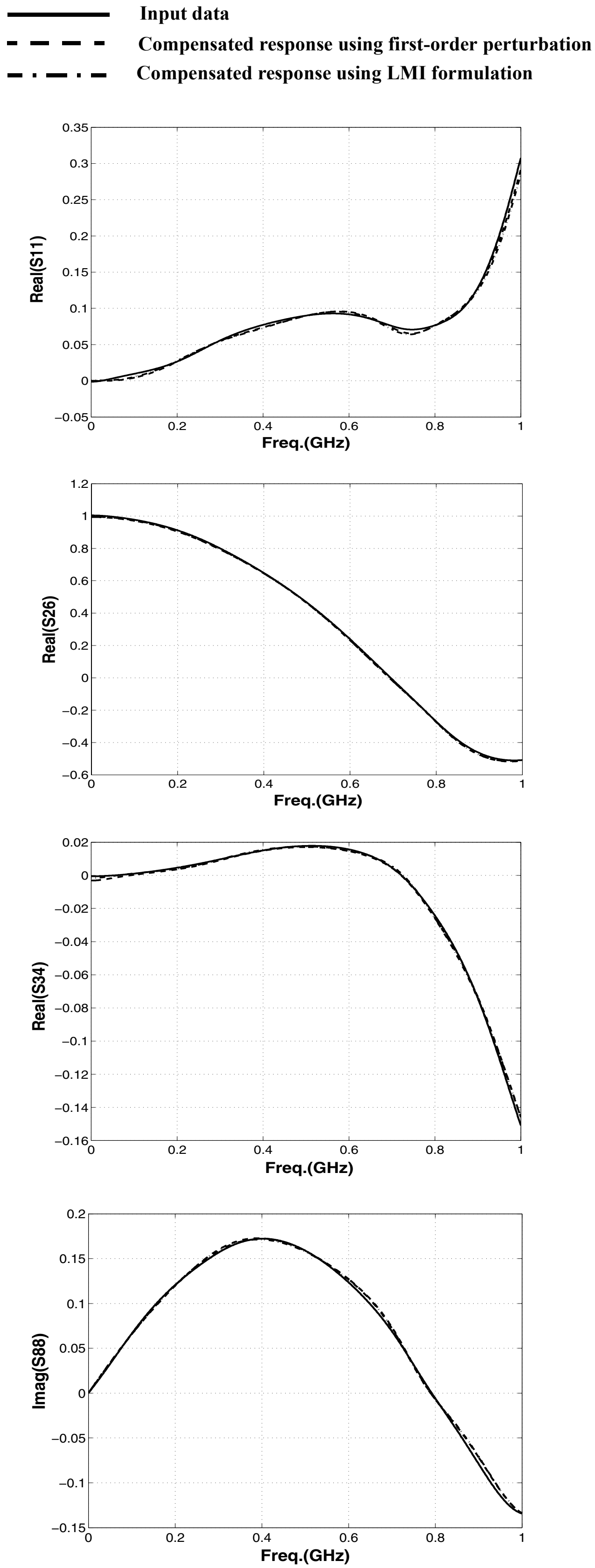

Fig. 1. S-parameters of RJ-45 connector

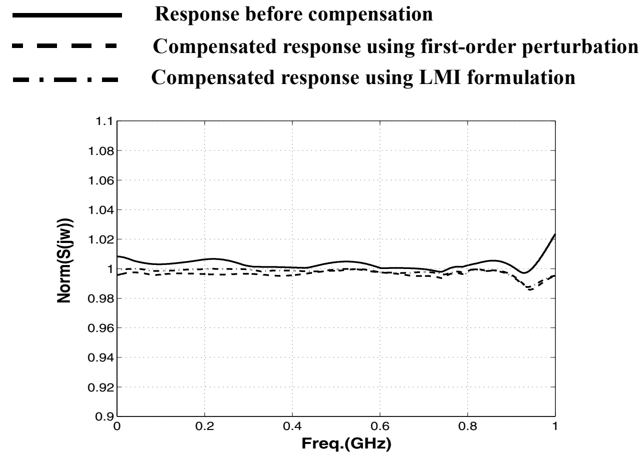

Fig. 2. Comparison of norm of $S(j\omega)$

VII. REFERENCES

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