Restoration of Passivity in S-parameter Data of Microwave Measurements

D. Saraswat, R. Achar and M. Nakhla

Department of Electronics, Carleton University, Ottawa, Ontario, K1S 5B6
Tel (613) 520-5651; Fax: (613) 520-5708; Email: achar@doe.carleton.ca

Abstract — **Circuit modeling of networks described by tabulated S-parameters has generated immense interest during the recent years. The tabulated data may be obtained either from measurements or full-wave EM simulations. However, one of the major difficulties with such type of data is that, the data can be non-passive in the frequency bandwidth of interest, due to the measurement errors or the numerical errors associated with the full-wave simulator. This causes significant difficulty while interfacing such a data with circuit simulators. To overcome this difficulty, this paper presents an efficient algorithm for restoring the passivity of the S-parameter data, prior to its circuit modeling. Numerical examples are presented to demonstrate the validity and efficiency of the proposed algorithm.**

Index Terms — **EM Subnetworks, Measured Data, Passive Macromodels, Scattering Parameters, Transient Analysis.**

I. INTRODUCTION

During the recent years, due to the continually increasing operating frequencies, circuit densities and complexities, characterization and simulation of high-speed microwave and electromagnetic subnetworks based on S-parameters has become a topic of intense research. Important applications of such a characterization include high-speed packages, vias and on-chip passive components [1]-[9], connectors, non uniform transmission lines etc. The S-parameters can be obtained either directly from measurements or from rigorous full-wave electromagnetic simulation. In the rest of the paper, we refer to such a data as the "*tabulated data*".

It is of practical interest to include above type of tabulated data during the global transient analysis of the entire system. However, interfacing such a data with circuit simulators for transient analysis purposes can suffer from the major difficulty, that the data can be non-passive in the frequency bandwidth of interest. Non-passivity of the data can be caused due to the measurement errors or the numerical errors associated with the electromagnetic analysis tools. It is to be noted

that, passivity is an important property, because stable but non-passive models may lead to unstable systems when connected to other passive devices. Implication of this from the transient analysis perspective can be explained as follows.

Transient analysis involving tabulated data can be mainly accomplished via convolution based approaches [8], [9] or passive macromodels [3]-[7]. If the convolution based approach is used, the frequency-domain S-parameter data is converted into time-domain using the IFFT algorithm. Subsequently, transient responses are evaluated by convoluting the IFFT response with the input responses. However this can run into convergence problems if the tabulated data is non-passive.

On the other hand, transient analysis using macromodels requires that the passivity of the macromodel is ensured, to guarantee asymptotic stability. Passive macromodeling algorithms available in the literature for this purpose are based on approximating the tabulated data through rational-functions and subsequently checking/ compensating for any passivity violation [3]-[7]. The time/effort spent in compensating the rational function model depends on the amount of the passivity violation in the rational-function fitted model. This in turn is influenced by the amount of the passivity violation present in the original tabulated data.

Hence, it is highly desired that the tabulated S-parameter data is made passive in the bandwidth of interest, prior to its circuit compatible modeling. In order to accomplish the above, this paper describes an efficient algorithm for restoring the passivity of the S-parameter data obtained from measurements or electromagnetic simulations. The method is based on the first-order matrix perturbation theory. Numerical examples are provided to validate the proposed algorithm. The pre-processed data using the new algorithm, allows it to be directly used for transient analysis using convolution techniques as well as results in significantly improved

performance for passive macromodeling algorithms.

The rest of the paper is organized as follows. Section II describes the problem definition, section III describes the proposed algorithm, while section IV and V present numerical results and conclusions, respectively.

II. PROBLEM DEFINITION

At higher frequencies, scattering parameters are widely used to characterize the complex interconnect and microwave subnetworks. S-parameter matrix (S) relate the incident travelling wave vectors (a) and the reflected travelling wave vectors (b) as

$$b = Sa \qquad (1)$$

The scattering parameters are obtained at several discrete frequency points in the bandwidth of interest. Let the highest frequency at which the data is given be denoted by w_{max}. The designers are interested in performing time-domain analysis including the device characterized by tabulated S-parameter subnetwork, along with the external terminations. As discussed in the introduction, passivity of the tabulated data becomes important for the success of passive macromodeling algorithms and to ensure the stability of the global transient simulation.

A network with scattering matrix $S(s)$ is passive [6] iff,

(a) $S(s^*) = S^*(s)$, '*' is the complex conjugate operator.

(b) $S(s)$ is bounded real, i.e. $\|S(j\omega)\|_2 \leq 1$ for $\omega \in \Re$ ($\| \ \|_2$ represents 2-norm).

Condition (b) also implies that $I - S^H S \geq 0$ for $\omega \in \Re$, where 'H' is the Hermitian operator.

Passive macromodeling algorithms strive to satisfy the above conditions for the entire frequency region $\omega \in (0 \to \infty)$, by employing various mechanisms. However, success of these algorithms depend on the passivity compliance of the original tabulated data (i.e. satisfying the conditions (a) and (b)) at the given discrete points in the region $\omega \in (0 \to w_{max})$). However, as pointed out in the introduction, the tabulated data in practice can be non-passive due to the measurement errors or numerical errors associated with the electromagnetic analysis tools. In the next section, an efficient algorithm for restoring the passivity of the non-passive tabulated data is described.

III. PROPOSED PASSIVITY RESTORATION ALGORITHM FOR S-PARAMETER DATA

The proposed passivity restoration algorithm is based on the following first order perturbation theory [10]:

<u>Lemma 1:</u> If a matrix Q is perturbed by an amount ΔQ, then the resulting change in its eigenvalue $(\Delta\lambda)$ is given by [10]:

$$\Delta\lambda = \frac{y^t \Delta Q x}{y^t x} \qquad (2)$$

where y and x are the left and right eigenvectors of Q, respectively.

Next, consider the S-parameter data at N discrete points. In the first step, the 2-norm of the S-parameter matrix is evaluated at each frequency point. The data corresponding to those frequency points, where the norm is greater than one (i. e., the data at these points violates the bounded real condition (b)) is collected in a set:

$$G = [S_k]; \quad k = 1 \ldots L \qquad (3)$$

where L represents the total number of non-passive data points.

Next, to illustrate the proposed passivity restoration algorithm, consider a kth non-passive S-parameter data matrix (i.e. $I - S_k^H S_k \leq 0$) in set G. The above also implies that, for this case, at least one of the eigenvalues of $I - S_k^H S_k$ is negative. Consider that an eigenvalue is negative by the amount $\Delta\lambda$. In the proposed algorithm, the data S_k is perturbed by an amount ΔS_k such that the perturbed data $S_k + \Delta S_k$ satisfies the following passivity criteria

$$I - (S_k + \Delta S_k)^H (S_k + \Delta S_k) \geq 0 \qquad (4)$$

Expanding the relationship (4) and neglecting the second order term of perturbation ΔS_k in it, implies that if we perturb $I - S_k^H S_k$ by $-S_k^H \Delta S_k - \Delta S_k^H S_k$, then from (2) it is obvious that the corresponding change in the eigenvalue of $I - S_k^H S_k$ is given by:

$$\Delta\lambda \;=\; \frac{\boldsymbol{v}^t(-\boldsymbol{S}_k^{H}\Delta\boldsymbol{S}_k - \Delta\boldsymbol{S}_k^{H}\boldsymbol{S}_k)\boldsymbol{u}}{\boldsymbol{v}^t\boldsymbol{u}} \qquad (5)$$

where $\boldsymbol{v}$ and $\boldsymbol{u}$ are the left and right eigenvectors of $\boldsymbol{I} - \boldsymbol{S}_k^{H}\boldsymbol{S}_k$, respectively. In other words, if the original data $\boldsymbol{S}_k$ is non-passive, then based on the knowledge of the extent of violation ($\Delta\lambda$), we compute a perturbation $\Delta\boldsymbol{S}_k$, such that the bounded realness relation (4) is satisfied.

Next, in order to evaluate the entries of the required perturbation matrix $\Delta\boldsymbol{S}_k$, (5) can be formulated as a least-square problem

$$\boldsymbol{WX} \;=\; \Delta\lambda \qquad (6)$$

where $\boldsymbol{X}$ is the vector of unknowns of the perturbation matrix $\Delta\boldsymbol{S}_k$, while $\boldsymbol{W}$ is composed of the entries from matrix $\boldsymbol{S}_k$, vectors $\boldsymbol{v}$ and $\boldsymbol{u}$. Equation (6) is solved at each entry of the non-passive data set $\boldsymbol{G}$. In order to minimize the perturbation in the data, (6) is solved with the constraint that $\|\boldsymbol{X}\|_2$ is minimized. It is to be noted that, solving (6) at a given frequency point (while minimizing $\|\boldsymbol{X}\|_2$) to correct a negative eigenvalue of $\boldsymbol{I} - \boldsymbol{S}_k^{H}\boldsymbol{S}_k$, may cause other eigenvalues to become negative. For the rare cases of this happening, in the proposed algorithm, the resulting S-parameter data is checked again for passivity violation and the above correction scheme is performed iteratively until the resulting data satisfies the passivity criteria (4).

Also, it is to be noted that, (5) is valid if the required perturbation $\Delta\boldsymbol{S}_k$ is relatively small. To ensure this for data with relatively large violations, we compensate for the violation $\Delta\lambda$ in steps of small increments, sequentially. In other words, if m steps are required to completely compensate the violation $\Delta\lambda$, total perturbation $\Delta\boldsymbol{S}_k$ is given by

$$\Delta\boldsymbol{S}_k \;=\; \sum_{p=1}^{m} \Delta\boldsymbol{S}_k^{p} \qquad (7)$$

where $\Delta\boldsymbol{S}_k^{p}$ is the perturbation at step number p.

IV. COMPUTATIONAL RESULTS

To demonstrate the validity and efficiency of the proposed algorithm, measurement data of a 2-port coaxial microstrip was considered. The tabulated S-parameters (henceforth referred to as the original data) were given up to 9GHz (sample responses are given in Fig.1 to Fig.5). As seen by the plot of $\|\boldsymbol{S}(j\omega)\|$ v/s frequency in Fig. 6 (solid line), the original data was not bounded-real at many frequency points.

Next, using the proposed algorithm of section III, the given S-parameter data was preprocessed to restore its bounded realness in the bandwidth of interest (this required only 15 seconds). As seen from the dashed line in Fig. 6, the bounded realness of the data was ensured in the bandwidth of interest (up to 9GHz). Next, the accuracy of the passivity restored data is verified by comparing them against the original data (see dashed lines in Fig.1 to Fig. 5). As can be seen from the plots, the proposed algorithm had negligible impact on the accuracy while restoring the passivity.

V. CONCLUSIONS

In this paper an efficient algorithm is presented for the passivity restoration of S-parameter tabulated data. The algorithm helps to restore the bounded-real property of the data in the bandwidth of interest. This allows the S-parameter data to be directly used for transient analysis using convolution techniques as well as results in significantly improved performance for passive macromodeling algorithms.

REFERENCES

[1] R. Achar, P. Gunupudi, M. Nakhla and E. Chiprout, "Passive Interconnect Reduction Algorithm for Distributed/Measured Networks", *TCAS-II*, pp. 287-301, April 2000.

[2] W. T. Beyene, and J. E. Schutt-Aine, "Accurate frequency-domain modelling and efficient simulation of high-speed packaging interconnects", *IEEE Transactions MTT*, pp. 1941-1947, Oct. 1997.

[3] D. Saraswat, R. Achar and M. Nakhla, "A Fast Algorithm and Practical Considerations For Passive Macromodeling Measured/Simulated Data", *IEEE Trans. on Advanced Packaging*, pp. 57-70, Feb. 2004.

[4] D.Saraswat, R. Achar and M. Nakhla, "Passive Macromodels of Microwave Subnetworks Characterized by Measured Data", *IMS-2003 Digest*, Philadelphia, PA, pp. 999-1002, June 03.

[5] D. Saraswat, R. Achar and M. Nakhla, "Fast Passive Macromodeling of S-parameter based Interconnect Subnetworks", *IEEE ANTEM*, pp.

357 – 360, July 2004.

[6] C. Coelho, J. Phillips, and L. Silveira, "Passive Constrained Rational Approximation using Nevanlinna-Pick Interpolation", *Proc. DATE*, June-02.

[7] C. P. Coelho, J. R. Phillips, and L. M. Silveira, "A convex programming approach to positive real rational approximation*", Proc. ICCAD*, pp. 245-251, Nov-01.

[8] A. R. Djordjević and T. K. Sarkar, "Analysis of lossy transmission lines with arbitrary nonlinear terminal networks", *IEEE Trans. Microwave Theory Tech.*, pp. 660-666, June 1986.

[9] J. E. Schutt-Aine and R. Mittra, "Scattering parameter transient analysis of transmission lines loaded with nonlinear terminations," *IEEE Trans. MTT*, vol. 36, pp. 529-536, 1988.

[10] G. W. Stewart and Ji-guang Sun, *Matrix perturbation theory,* Publisher Boston: Academic Press, 1990.

[11] MATLAB version 6.5.0.180913a, Release 13, June 2002.
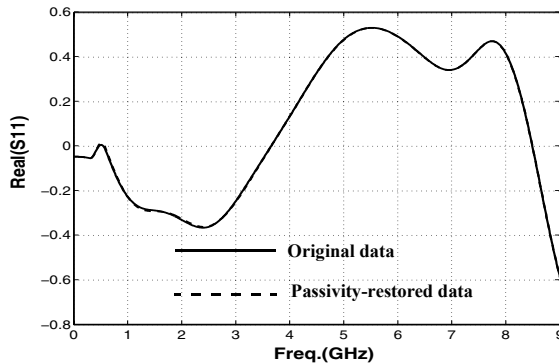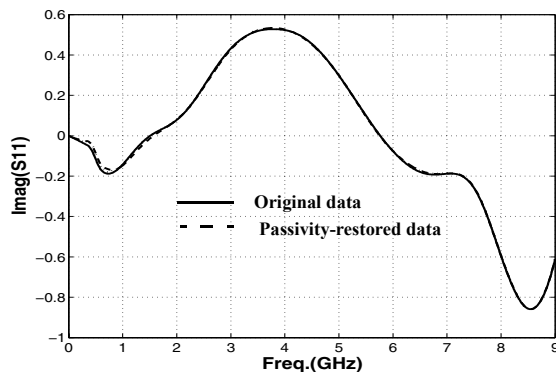
Fig. 1. Real (S_{11}) v/s Frequency

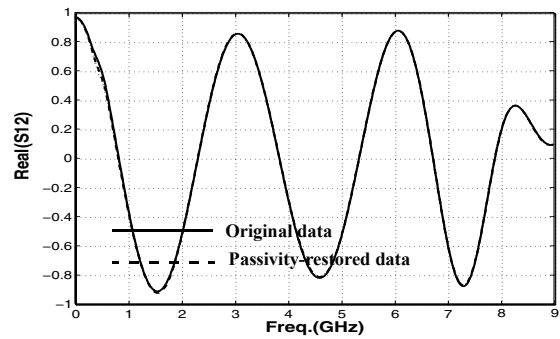Fig. 2. Imag (S_{11}) v/s Frequency
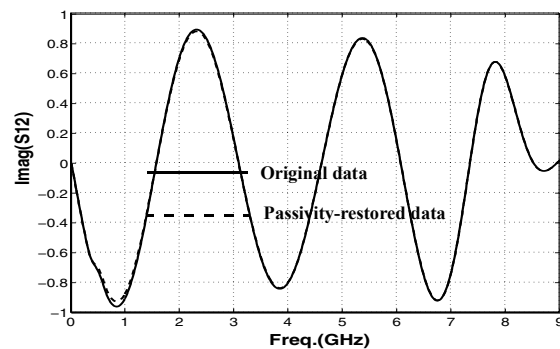

Fig. 3. Real (S_{12}) v/s Frequency

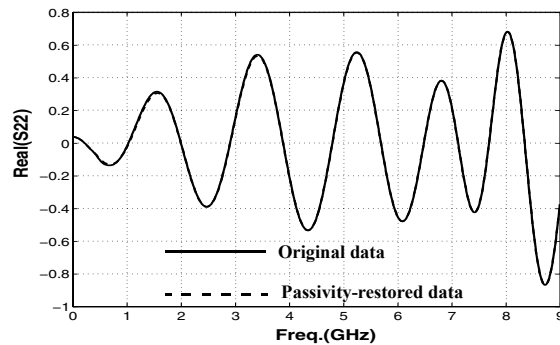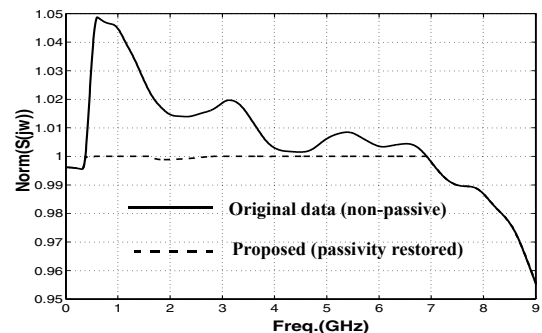Fig. 4. Imag (S_{12}) v/s Frequency

Fig. 5. Real (S_{22}) v/s Frequency

Fig. 6. Norm of the data before and after passivity restoration