PRIME: PASSIVE REALIZATION OF INTERCONNECT MODELS FROM MEASURED DATA

Jason Morsey and Andreas C. Cangellaris
ECE Department, University of Illinois at Urbana-Champaign
1406 W. Green Street, Urbana, IL 61801-2991
Phone: 217.333.6037; Fax: 217.333.5962; E-mail: {morsey, cangella}@uiuc.edu

Abstract

A methodology is presented for synthesis of passive, broadband equivalent circuits for interconnect multi-ports from measured scattering-parameter data. In addition to the mathematical details of the synthesis process, results from its application to the synthesis of equivalent circuits for multi-GHz bandwidth connectors are used to demonstrate the validity and accuracy of the proposed methodology.

Introduction

With off-chip signal bandwidths in the order of tens of GHz in state-of-the-art and future digital electronic systems, accurate modeling of connectors and other types of high-complexity interfaces present in the packaging hierarchy is essential for signal integrity analysis and global interconnect electrical performance assessment. However, it is often the case that, due to their high density and geometric complexity, the extraction of the electromagnetic response for such structures through computer modeling is rather tedious and often times computationally prohibitive. In such cases, the development of electrical models through measurements is the preferred alternative.

Following such an experimental characterization procedure, the synthesis of an equivalent circuit model for the measured component is a necessary step toward the realization of a SPICE-compatible electrical model to enable global interconnect electrical performance assessment through transient simulation. Several methodologies have been presented for the synthesis of such equivalent circuits and the cited references [1]-[3] offer a very small sample of the relevant literature most closely related to high-speed interconnect and packaging structures. In all cases, the primary challenge is to ensure the passivity of the synthesized equivalent circuit over the entire bandwidth of interest, which tends to be in the order of tens of GHz for many of the state-of-the-art performance-driven digital and opto-electronic systems.

The proposed methodology in this paper is aimed at providing a straightforward approach to such synthesis. It consists of two steps. The first step involves the fitting of the measured data into a pole-residue form, subject to the requirement that the same set of poles is being used for all the elements in the scattering parameter matrix [4]. This step is common to most of the fitting processes used today and will not be elaborated in this paper. The second step involves the post-processing of the synthesized matrix pole-residue representation of the measured component in order to ensure its realization in terms of a passive circuit. The way this is done is presented next.

Passive Realization

For the purposes of this paper the equivalent circuit synthesis is based on the fitted admittance matrix for the multi-port. The admittance matrix parameters for a multi-port are obtained from its scattering parameters by means of well-known relationships [5]. Let N be the number of poles in the fitted multi-port admittance matrix. For stability, the poles, a_n, $n = 1, 2, \ldots, N$, must have non-negative real part. This requirement is imposed as a constraint in the fitting process. Consequently, the result of the fitting may be cast in the following form,

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$$\mathbf{Y}(s)=\mathbf{A}^{(0)}+\mathbf{A}^{(\infty)}s+\sum_{n=1}^{N}\left(\frac{A_0^{(n)}}{s-\alpha_n}+\frac{\bar{A}_0^{(n)}}{s-\bar{\alpha}_n}\right)\mathbf{A}^{(n)} \tag{1}$$

where the notation $\bar{z}$ denotes the complex conjugate of z. This expression is recognized as Foster's canonical representation. Following [6] for the extension of Foster's canonical representation to the case of lossy, reciprocal multi-ports, and using the result that the admittance matrix for a passive multi-port is a positive real matrix function [7], the matrices $\mathbf{A}^{(n)}$ in (1) must be real, symmetric, and positive semi-definite. However, the presence of noise in the measurement process as well numerical noise in the fitting process are responsible for the absence of these attributes from the set of matrices obtained immediately after fitting. Thus, a post-processing of the generated form in (1) is required to render the matrices $\mathbf{A}^{(n)}$ real, symmetric, and positive semi-definite. This is done in the following manner.

Without loss of generality, consider the case of a complex conjugate pair of poles. Assuming that the number of ports is M, the associated pole-residue term is of the form,

$$\left(\frac{A_0^{(n)}}{s-\alpha_n}+\frac{\bar{A}_0^{(n)}}{s-\bar{\alpha}_n}\right)\mathbf{A}^{(n)}=\left(\frac{A_0^{(n)}}{s-\alpha_n}+\frac{\bar{A}_0^{(n)}}{s-\bar{\alpha}_n}\right)\begin{bmatrix} r_{11}^{(n)} & r_{12}^{(n)} & \cdots & r_{1M}^{(n)} \\ r_{21}^{(n)} & r_{22}^{(n)} & \cdots & r_{2M}^{(n)} \\ \vdots & \vdots & \ddots & \vdots \\ r_{M1}^{(n)} & r_{M2}^{(n)} & \cdots & r_{MM}^{(n)} \end{bmatrix} \tag{2}$$

The $\mathbf{A}^{(n)}$ matrix in (2) must be real, symmetric and positive semi-definite. Following the eigen-decomposition $\mathbf{A}^{(n)}=\mathbf{V}diag(\lambda_1,\lambda_2,\ldots,\lambda_M)\mathbf{V}^{-1}$, all eigenvalues with negative real part are set to 0. Any remaining complex eigenvalues are made real by replacing them with their magnitude. Subsequently, the matrix is reconstructed through the operation $\tilde{\mathbf{A}}^{(n)}=\mathrm{Re}\{\mathbf{V}diag(\tilde{\lambda}_1,\tilde{\lambda}_2,\ldots,\tilde{\lambda}_M)\mathbf{V}^{-1}\}$ where the modified quantities are denoted with "~".

The next step concerns the enforcement of the conditions required for the elements of the circuits that are used to synthesize the term

$$\frac{A_0^{(n)}}{s-\alpha_n}+\frac{\bar{A}_0^{(n)}}{s-\bar{\alpha}_n} \tag{3}$$

to be real and non-negative. As shown in [6], the term in (3) can be synthesized through an RLCG equivalent circuit of with element values given by the following relationships,

$$L^{-1}=2\mathrm{Re}\{A_0^{(n)}\},\quad GC^{-1}=\frac{-\mathrm{Re}\{A_0^{(n)}\bar{\alpha}_n\}}{\mathrm{Re}\{A_0^{(n)}\}},\quad RL^{-1}=\frac{\mathrm{Re}\{A_0^{(n)}\bar{\alpha}_n\}}{\mathrm{Re}\{A_0^{(n)}\}}-2\mathrm{Re}\{\alpha_n\} \tag{4}$$

Considering that the real part of the pole is negative or zero, the right-hand sides of the above three equations are non-negative if

$$\mathrm{Re}\{A_0^{(n)}\}>0 \text{ and } 0\le \mathrm{Im}\{A_0^{(n)}\}\mathrm{Im}\{\alpha_n\}\le\left|\mathrm{Re}\{A_0^{(n)}\}\mathrm{Re}\{\alpha_n\}\right| \tag{5}$$

The second condition in (5) is checked and, if violated, $A_0^{(n)}$ is made real by setting its imaginary part to zero. Next, the real part of $A_0^{(n)}$ is examined and, if non-positive, the pole pair and associated residue are dropped.

Following the completion of this post-processing step, the adjusted admittance matrix satisfies all requirements for it to be the canonical Foster form for a linear, reciprocal and passive multi-port. The subsequent synthesis of the equivalent circuit is straightforward. Instead of the rather involved realization described in [6] in terms of RLCG lumped elements and transformers, controlled sources instead of transformers can be used to simplify the form of the equivalent circuit and facilitate its synthesis.

Numerical Implementation

In order to test the accuracy of PRIME, the algorithm was performed on measured scattering parameters for a single-pin connector. The connector was measured at 561 frequency points, equally spaced over a bandwidth of 35GHz. The admittance parameters for this 2 port were obtained from the measured data and fitted using the vector-fitting algorithm VECTFIT [8] with $\mathbf{A}^{(0)}$ and $\mathbf{A}^{(\infty)}$ in equation (1) equal to zero. The approximation used 50 stable poles, two of which were removed during post-processing. The final approximation used two real poles and 23 conjugate pairs, and had an rms error of 0.0046 S. A SPICE equivalent circuit was then generated.

Figure 1 shows the agreement for Y_{12} between the measured data and the values obtained by simulating the equivalent circuit in SPICE. To verify the passivity of the circuit, a transient analysis was performed. The circuit was excited with a trapezoidal pulse train at port 1with a source impedance of 50 ohms. The pulse train cycled between 0 and 3.3 volts with a period of 500psec, a 50% duty cycle, and a rise and fall time of 50psec. Port 2 was terminated with a 1pF capacitor. Figure 2 shows the voltage waveforms at the source and load. It is important to note that the load voltage remains bounded, showing that the approximation is indeed passive.

Conclusion

In summary, a new approach, labeled PRIME, has been introduced for converting from measured scattering or admittance parameters to passive SPICE-compatible circuits. This new approach outlines the method for taking a general pole-residue representation and reducing it to a passive representation. The new representation is then modeled with basic circuit elements and dependent sources. This circuit can then be used for accurate time domain simulations over multi-GHz bandwidths. A demonstration of the PRIME algorithm was performed on data from a measured two-port network showing its accuracy and passivity over a 35GHZ bandwidth.

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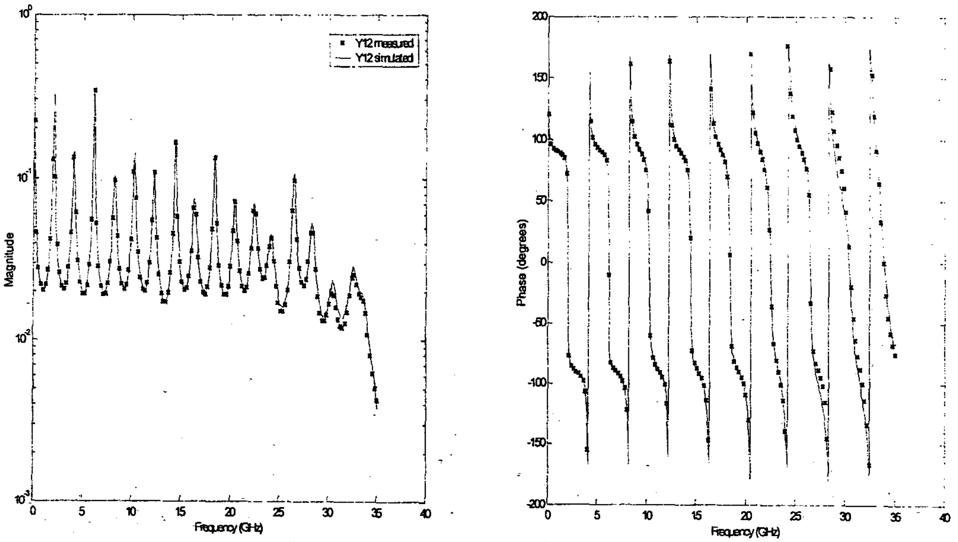

Figure 1. Comparison of measured admittance parameter Y_{12} and those simulated with SPICE circuit for single-pin connector. The "measured" data was obtained by transforming the measured scattering parameters to admittance parameters.

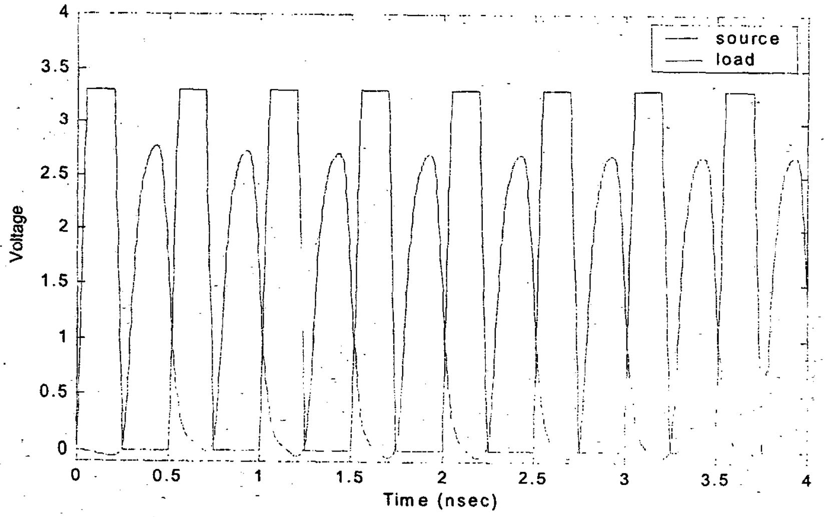

Figure 2. Transient analysis of single-pin connector using SPICE generated circuit.