Weekly Report for Yu Hu's work from Sep. 26 to Oct. 2

October 2, 2005

1 Clean Yan's bottom-up vdd assignment and global refinement code

2 Search and read literature about robust optimization and process variation

3 Formulate the dual-vdd assignment with process variation problem to network flow

The original dual-vdd assignment (with mixed wire length) problem is presented as follows [1].

$$Maximize \sum_{i=0}^{N_r-1}\sum_{j=0}^{N_s(i)-1} \Delta P_d(i,j) f_n(i,j) + \sum_{i=0}^{N_{r-1}}\sum_{j=0}^{N_s(i)-1} f_n(i,j)\Delta P_s(i,j) \quad (1)$$

$$s.t.$$

$$f_n(i,j) \le \frac{s_{ik}}{C_{ik}} c_{ij} \qquad 0 \le i < N_r \wedge 0 \le j < N_s(i) \wedge \forall k \in \mathcal{SL}_{ij} \quad (2)$$

$$a(v) \le T_{spec} \qquad \forall v \in \mathcal{PO} \quad (3)$$

$$a(v) = 0 \qquad \forall v \in \mathcal{PI} \quad (4)$$

$$a(p_{i0}) + d(p_{i0}, p_{ik}) + S_{ik} \le a(p_{ik}) \quad (5)$$

$$0 \le i < N_r \wedge \forall p_{ik} \in \mathcal{FO}_{p_{i0}} \quad (6)$$

$$0 \le S_{ik} \le D_{ik} \qquad 0 \le i < N_r \wedge \le k \le N_k(i) \quad (7)$$

$$s_{ik} = \frac{S_{ik}}{D_{ik}} \cdot l_{ik} \quad (8)$$

$$(9)$$

where,

$$\Delta P_d(i,j) = c_{ij} f_{clk}[\Delta E_0(i,j) + c_4(i,j)V] \quad (10)$$

$$\Delta P_s(i,j) = \Delta P_0 e^{-c_1(i,j)L - c_2(i,j)V - c_3(i,j)T} \quad (11)$$

We need the following three steps to reformulate this problem to a min-cost network flow problem: 1) get the robust LP formulation of the original problem, 2) get the approximation LP counterpart of the robust LP formulation (according to the timing/power yield), 3) get the dual problem of the approximation LP, which is a min-cost flow problem.

3.1 Robust LP formulation of the original problem

As the same way in yu_sep18.pdf, we can re-write the above formulation by removing min:

[1] Denotations can be found in http://eda.ee.ucla.edu/member_only/FPGA_reports/rlp_slack.pdf

$$\begin{array}{ll} \max & \sum_{i=0}^{N_r-1}\sum_{j=0}^{N_k(i)-1} W_{ij}\cdot S_{ij} = \sum_{\forall Sink} W_{ij}\cdot S_{ij} \\ s.t. & a(j)-a(i) \le -d(i,j) \\ & a(i)-a(j) \le u_{ij} = l_{ij}D_{ij} + d(i,j) \\ & S_{ij} = a(i)-a(j)-d(i,j) \end{array} \tag{12}$$

where,

$$W_{ij} = \sum_{\forall k\in \mathcal{UBC}_{ij}} \{c_{ik} f_{clk} f_s(i,k)[\Delta E_0(i,k) + c_4(i,k)V] + \Delta P_0 e^{-c_1(i,k)L - c_2(i,k)V - c_3(i,k)T}\} \cdot \frac{c_{ik}D_{ij}}{(C_{ij}l_{ij})} \tag{13}$$

where i, j, k refers to routing tree i, sink j and buffer k in tree i.

Similar to [Murari et al DAC'05], we re-write the above formulation as follows. P^* is the optimal power achieved by deterministic version of formulation 12, and P_{max} is the initial maximum power.

$$\min \quad \sum_{\forall Sink} S_{ij} \tag{14}$$

$$s.t. \quad \sum_{\forall Sink} W_{ij}\cdot S_{ij} \ge P_{max} - P^* \tag{15}$$

$$a(j)-a(i) \le -d(i,j) \tag{16}$$

$$a(i)-a(j) \le u_{ij} = l_{ij}D_{ij} + d(i,j) \tag{17}$$

$$S_{ij} = a(i)-a(j)-d(i,j) \tag{18}$$

We then decompose the power reduction into each sink of each routing tree, and re-write Eq.15 as

$$\begin{array}{lll} & W_{ij}\cdot S_{ij} \ge \Delta P_{ij} & \forall Sink_j \in Net_i \\ \Longrightarrow & S_{ij} \ge \Delta P_{ij}/W_{ij} & \forall Sink_j \in Net_i \end{array} \tag{19}$$

With substituting Eq.18 into Eq.15 and Eq.14, then our formulation becomes

$$\begin{array}{lll} \min & \sum_{\forall V} \rho_i a_i & \\ s.t. & a(j)-a(i) \le d(i,j) - \Delta P_{ij}/W_{ij} & \forall Sink_j \in Net_i \\ & a(j)-a(i) \le -d(i,j) & \forall edge(i,j) \\ & a(i)-a(j) \le l_{ij}D_{ij} + d(i,j) & \forall edge(i,j) \end{array} \tag{20}$$

where $\rho_i = out(v_i) - in(v_i)$.

Note that Formulation 20 is a robust LP optimization problem, in which, S_{ij} and $a(i)$ are variables, W_{ij} and $d(i,j)$ are uncertain coefficients. W_{ij} captures the affect of process variation to both dynamic power and leakage power, and $d(i,j)$ captures those to delay. Particularly, we have $W_{ij} = f_w(L,V,T)$ and $d(i,j) = f_d(L,V)$, where L,V,T are the gate length, V_{th} and T_{ox} respectively.

3.2 Approximation of the robust LP formulation (still a LP)

THEOREM (Calafiore, Campi, 2002): Given a robust convex problem

$$\min cx : \forall u \in \mathcal{U}, f_i(x,u) \le 0, i = 1,\cdots,m \tag{21}$$

where $f_i(x,u)$ is convex and $\mathcal{U}$ is compact. We can replace $\mathcal{U}$ by a randomly chosen finite subset of $\mathcal{U}$ and solve corresponding convex problem. If the number of samples satisfies

$$N \ge \frac{n}{\epsilon\beta} - 1 \tag{22}$$

then with probability $1-\beta$, the probability of violation of the constraints is less than ϵ.

Obviously, the range of process variation parameters L, V, T can be limited within a compact set. Based this theorem, we can perform sampling (e.g. random generate a N-length vector (L, V, T) with a normal distribution) in the all the possible values of L, V, T. The number of samples can be decided by the required timing/power yield rate. For each sample of (L_s, V_s, T_s), we can get $W_{ij}^s = f_w(L_s, V_s, T_s)$ and $d^s i, j = f_d(L_s, V_s)$, which adds a new deterministic constraint in formulation 20. The union of all these new constraints makes the ϵ approximation of the formulation 20.

On the other hand, we can find that the increment of the sampling number won't increase number of constraints in Formulation 20 due to its special structure (all uncertain coefficient is not a multiplier of variables, when we get the union of all constraints, there will be only one constraint according to each W_{ij} and $d(i, j)$. e.g. for each sampled values of W_{ij}, only the minimum one will be used in the formula). Based on these, after sampling and union constraints, the ϵ approximation of formulation 20 is

$$\begin{array}{ll} \min & \sum_{\forall V} \rho_i a_i \\ s.t. & a(j) - a(i) \leq L_{ij} \\ & a(i) - a(j) \leq U_{ij} \quad \forall edge(i, j) \end{array} \tag{23}$$

where $U_{ij} = l_{ij} D_{ij} + d^s(i, j)$, and

$$L_{ij} = \{ \begin{array}{ll} \min(d^s(i, j) - \Delta P_{ij}/W_{ij}^s, -d^s(i, j)) & \forall Sink_j \in Net_i \\ -d^s(i, j) & \forall edge(i, j) \bar{\in} Sink \end{array} \tag{24}$$

3.3 The dual of the approximation LP formulation is a min-cost flow problem

The dual problem of formulation 23 is

$$\min \quad \sum_{e_{ij} \in E} -U_{ij} z_{ij} - L_{ij} y_{ij} \tag{25}$$

$$s.t. \quad \sum_{e_{ki} \in E} (y_{ki} - z_{ki}) - \sum_{e_{ij} \in E} (y_{ij} - z_{ij}) = \rho_i \tag{26}$$

$$y_{ij}, z_{ij} \in R_+ \tag{27}$$

This is a min-cost network flow problem and can be solved efficiently.