Weekly Report for Yu Hu's work from Sep. 17 to Sep. 18

September 19, 2005

I rewrite the LP formulation in Yan's DAC'05 paper to network flow. The original problem is as follows.

$$\max \sum_{i=0}^{N_r-1} (0.5 f_{clk} \cdot c \cdot \Delta V_{dd}^2 \cdot f_s(i) + \Delta P_s) \cdot F_N(i) \tag{1}$$

s.t.

$$F_N(i) = \textstyle\sum_{j=0}^{N_s(i)-1} f_N(i,j) \quad 0 \le i < N_r \tag{2}$$
$$f_N(i,j) \le s(i,k)/l(i,k) \quad \forall k \in \mathcal{SL}_{ij} \tag{3}$$

$$a(v) \le T_{spec} \quad \forall v \in \mathcal{PO} \tag{4}$$
$$a(v) = 0 \quad \forall v \in \mathcal{PI} \tag{5}$$
$$a(p_{i0}) + d(p_{i0}, p_{ik}) + s_{ik} \cdot \Delta d \le a(p_{ik}) \quad 0 \le i < N_r \land \forall p_{ik} \in \mathcal{FO}_{p_{i0}} \tag{6}$$
$$a(v) + d(u,v) \le a(v) \quad \forall u \in \mathcal{V} \land u \notin \mathcal{SRC} \land v \in \mathcal{FO}_u \tag{7}$$
$$0 \le s_{ik} \le l_{ik} \quad 0 \le i < N_r \land 1 \le k \le N_k(i) \tag{8}$$

Here I omitted all explanations for the denotations, which are the same with Yan's DAC'05 paper. In fact, F_n in Eq. 1 is

$$F_n(i) = \sum_{j=0}^{N_s(i)-1} \min(\frac{s_{ik}}{l_{ik}} : \forall k \in \mathcal{SL}_{ij}) \tag{9}$$

I rewrite Eq. 9 as

$$F_n(i) = \sum_{\forall b \in \mathcal{NSB}} \frac{s_{ik}^b}{l_{ik}} + \sum_{\forall p \in \mathcal{SB}} \frac{s_{ic}^p}{l_{ic}} \tag{10}$$

where set $\mathcal{NSB}$ include all buffers who are not shared by more than one sinks, while set $\mathcal{SB}$ include all buffers who are shared by more than on sinks, s_{ic}^p is the most critical sink under the fanout cone in shared buffer i in routing tree $\mathcal{R}_i$.

Obviously, based on Eq.10, which is a heuristic lower bound of the number of low-V_{dd} buffer can be achieved within a routing tree, we can draw a linear relationship between low-V_{dd} buffer number and slack. Therefore, for each sink s_{ij} in routing tree with source Src_i, we can calculate a weight W_{ij} based on Eq.10.

$$W_{ij} = \{ \begin{array}{ll} (0.5 f_{clk} \cdot c \cdot \Delta V_{dd}^2 \cdot f_s(i) + \Delta P_s) \cdot \sum_{\forall k \in \mathcal{UBC}_{ij}} 1/l_{ij} & e(i,j) \in \mathcal{R} \\ 0 & otherwise \end{array} \tag{11}$$

where set $\mathcal{UBC}$ include all buffers who are at the upstream of sink j at routing tree i with sink j as its most critical sink, and $\mathcal{R}$ is the set of routing trees.

Therefore, based on Majid's ICCAD'04 paper, we can write this problem as a min-cost flow problem. Firstly, we need to do the following modifications on the existing formulation.

1. Objective function is rewritten as

$$\max \sum_{i=0}^{N_r-1} \sum_{j=0}^{N_k(i)-1} W_{ij} \cdot s_{ij} = \sum_{\forall Sink} W_{ij} \cdot s_{ij} \tag{12}$$

2. Remove Eq.2 and Eq.3.

3. Add two virtual nodes S and T in timing graph, and add an edge from T to S with delay $-T_{spec}$, so that we can remove Eq.4 and Eq.4.

4. Based on lemma 1 in Majid's ICCAD'04 paper, we can rewrite Eq.6 as

$$a(p_{i0}) + d(p_{i0}, p_{ik}) + s_{ik} \cdot \Delta d = a(p_{ik}) \tag{13}$$

then we have

$$a(i) = a(j) + d(i,j) + s_{ij}\Delta d \tag{14}$$

generally, we use $a(i)$ and $a(j)$ to denote two nodes in timing graph then

$$s_{ij} = \frac{a(i) - a(j) - d(i,j)}{\Delta d} \tag{15}$$

5. substitute s_{ij} in Eq.8, we have

$$a(j) - a(i) \le -d(i,j) \tag{16}$$

and

$$\frac{a(i) - a(j) - d(i,j)}{\Delta d} \le l_{ij} \qquad \Rightarrow \qquad a(i) - a(j) \le u_{ij} = l_{ij}\Delta d + d(i,j) \tag{17}$$

6. substitute s_{ij} in Eq.12, we have the new objective function.

$$\max \sum_{e(i,j)\in E} (a(i) - a(j) - d(i,j))W_{ij} = \sum_{v_i \in V} a(i)\Big(\sum_{v_j \in out(v_i)} W_{ij} - \sum_{v_k \in in(v_i)} W_{ki}\Big) - \sum Wd \tag{18}$$

where $\sum Wd$ is a constant, so we can leave it out of the objective function.

Finally, we get the new primal problem formulation with Eq.18 as objective function, Eq.16 and Eq.4 as constraints. The dual problem can be written as follow.

$$\min \sum_{e_{ij}\in E} u_{ij}z_{ij} - d(i,j)y_{ij} \tag{19}$$

$$\sum_{e_{ki}\in E} (y_{ki} - z_{ki}) - \sum_{e_{ij}\in E} (y_{ij} - z_{ij}) = \rho_i \tag{20}$$

$$\rho i = \sum_{v_j \in out(v_i)} W_{ij} - \sum_{v_k \in in(v_i)} W_{ki} \tag{21}$$

$$y_{ij}, z_{ij} \in Z_+ \tag{22}$$

This is a min-cost flow problem as described in Majid's ICCAD'04 paper, and we can then follow their steps to solve the problem. Currently, I'm working on the coding and it'll take about two more days to get some primary results.