# Paper under Review - Please DO NOT Distribution Power-optimal Repeater Insertion Considering Vdd and Vth as Design Freedoms 

Yu Ching Chang<br>University of California, Los Angeles, CA 90095, USA<br>ychangu@ee.ucla.edu

King Ho Tam<br>University of California, Los Angeles, CA 90095, USA<br>ktam@ee.ucla.edu

Lei He<br>University of California, Los Angeles, CA 90095, USA<br>Ihe@ee.ucla.edu


#### Abstract

This work first presents an analytical repeater insertion method which optimizes power under delay constraint for a single net. This method finds the optimal repeater insertion lengths, repeater sizes, and $V_{d d}$ and $V_{t h}$ levels for a net with a delay target, and it reduces more than $50 \%$ power over a previous work which does not consider $V_{d d}$ and $V_{t h}$ optimization. This work further presents the power saving when multiple $V_{d d}$ and $V_{t h}$ levels are used in repeater insertion at the full-chip level. Compared to the case with single $V_{d d}$ and $V_{t h}$ suggested by ITRS, optimized dual $V_{d d}$ and dual $V_{\text {th }}$ can reduce overall global interconnect power by $47 \%, 28 \%$ and $13 \%$ for $130 \mathrm{~nm}, 90 \mathrm{~nm}$ and 65 nm technology nodes, respectively, but extra $V_{d d}$ or $V_{t h}$ levels only give marginal improvement. We also analyze the trends of $V_{d d}$ and $V_{t h}$ optimization for chip level power reduction, and show that an optimized single $V_{t h}$ can reduce interconnect power almost as effective as dual $-V_{t h}$ does.


## 1. INTRODUCTION

Repeater insertion is extensively used in nowadays designs for delay reduction in long interconnect, which causes increasingly severe problem of power consumption due to the ever increasing number of repeaters [1]. Traditional approach of repeater insertion optimizes the interconnect in terms of delay, but several works in the literature $[2,3,4]$ have made use of the extra tolerable delay (i.e., slack) in nets for significant saving in interconnect power. [2, 3] provide analytical methods to compute unit length power optimal repeater insertion solutions. [4] defines a new figure of merit which allows trade-off between power and delay using repeater insertion legnths, repeater sizes and wire widths as design knobs. None of the above work consider supply voltage $V_{d d}$ and threshold voltage $V_{t h}$ as design freedoms. [5] performs dual $V_{d d}$ and dual $V_{t h}$ assignments on logic circuits to reduce power consumption, and shows that $20 \%$ of power can be saved by going from single $V_{t h}$ to dual $V_{t h}$ under the dual $V_{d d}$ power supply.

This paper studies the opportunity of power saving by computing power optimal repeater sizes, repeater insertion lengths, and for the first time $V_{d d}$ and $V_{t h}$ levels for both individual nets and full chips. Our first contribution derives a set of analytical formulae which finds the optimal interconnect power given the amount of the timing slack on a single net. Our results show that more than $50 \%$ of power saving can be achieved over [2] which does not consider $V_{d d}$ and $V_{t h}$ as design variables. Our second contribution studies the power saving of using multiple $V_{d d}$ and $V_{t h}$ levels for buffering interconnects. Compared to the case without $V_{d d}$ and $V_{t h}$
optimization, optimized dual $V_{d d}$ and dual $V_{t h}$ can reduce overall global interconnect power by $47 \%, 28 \%$ and $13 \%$ for $130 \mathrm{~nm}, 90 \mathrm{~nm}$ and 65 nm technology nodes, respectively, but extra $V_{d d}$ or $V_{t h}$ level only gives marginal improvement. We also analyze the trends of $V_{d d}$ and $V_{t h}$ optimization for chip level power reduction, and show that an optimized single $V_{t h}$ can reduce interconnect power almost as effective as dual- $V_{t h}$ does.

This paper is organized as follows. Section 2 discusses the delay and the power models. Section 3 presents single-net power optimization with $V_{d d}$ and $V_{t h}$ tuning. Section 4 studies the full chip power optimization using multiple $V_{d d}$ and $V_{t h}$. We conclude in Section 5.

## 2. PRELIMINARIES

This section discusses the delay and power models used in this paper. Both the delay and power models are based on those in [2], which assume fixed $V_{d d}$ and $V_{t h}$. We extend the models to reflect the effects of $V_{d d}$ and $V_{t h}$ scaling.

### 2.1 Delay Model

Consider an interconnect segment of unit length resistance $r$ and unit length capacitance $c$. It is driven by a repeater of size $s$ with unit driving resistance $r_{s}$, unit input capacitance $c_{p}$ and unit output capacitance $c_{o}$. We assume that the interconnect is terminated at the other end with another repeater of identical size. Suppose the interconnect segment is of length $l$, the delay of the driving repeater and the wire segment is

$$
\begin{equation*}
\tau=r_{s}\left(c_{o}+c_{p}\right)+\frac{r_{s}}{s} c l+r l s c_{o}+\frac{1}{2} r c l^{2} \tag{1}
\end{equation*}
$$

and the unit length delay is

$$
\begin{equation*}
\frac{\tau}{l}=\frac{1}{l} r_{s}\left(c_{o}+c_{p}\right)+\frac{r_{s}}{s} c+r s c_{o}+\frac{1}{2} r c l \tag{2}
\end{equation*}
$$

In Equation (2), the driving strength of a repeater depends on the operating $V_{d d}$ and $V_{t h}$ levels and the driving resistance can be approximated in [3] by

$$
\begin{equation*}
r_{s}=K_{1} \frac{V_{d d}}{I_{d s a t}} \tag{3}
\end{equation*}
$$

where $K_{1}$ is a fitting parameter and $I_{d s a t}$ is the saturated drain current of a minimum-sized NMOS or PMOS transistor with both $V_{g s}$ and $V_{d s}$ equal to $V_{d d}$. According to the alpha-power law model [6], $I_{d s a t}$ is modeled as

$$
\begin{align*}
I_{d s a t} & =K_{2}\left(V_{g s}-V_{t h}\right)^{\alpha} \\
& =K_{2}\left(V_{d d}-V_{t h}\right)^{\alpha} \tag{4}
\end{align*}
$$

where $K_{2}$ is a device parameter and $\alpha$ typically equals to 1.25 . By plugging Equation (4) into Equation (3), we obtain $r_{s}$ as a function of $V_{d d}$ and $V_{t h}$.

$$
\begin{equation*}
r_{s}=K_{3} \frac{V_{d d}}{\left(V_{d d}-V_{t h}\right)^{\alpha}} \tag{5}
\end{equation*}
$$

where $K_{3}=K_{1} / K_{2}$. For a given $V_{d d}$ and $V_{t h}$, the unit length delay is optimal when

$$
\begin{equation*}
l_{o p t}=\sqrt{\frac{2 r_{s}\left(c_{o}+c_{p}\right)}{r c}} \quad s_{o p t}=\sqrt{\frac{r_{s} c}{r c_{o}}} \tag{6}
\end{equation*}
$$

which results in the optimum unit length delay given by

$$
\begin{equation*}
\left(\frac{\tau}{l}\right)_{o p t}=2 \sqrt{r_{s} c_{o} r c}\left(1+\sqrt{\frac{1}{2}\left(1+\frac{c_{p}}{c_{o}}\right)}\right) \tag{7}
\end{equation*}
$$

When the delay target is larger than $\left(\frac{\tau}{l}\right)_{\text {opt }}$, we can find a family of solutions $\left\{V_{d d}, V_{t h}, l, s\right\}$ that satisfy the target delay [2]. In the solution set, there exists a solution that achieves the minimum power. The methodology of finding such solution is presented in Section 3.

### 2.2 Power Model

The power dissipation of a repeater comprises three parts: dynamic, leakage, and short circuit. We use the same formulae to compute power as in [2] except that $V_{d d}$ and $V_{t h}$ are taken out of the constant coefficients and are treated as variables in the expressions. The power models are summarized below.
Dynamic power is the power dissipated when repeaters charge and discharge their loading capacitances. It is given by

$$
P_{\text {switching }}=a\left(s\left(c_{o}+c_{p}\right)+l c\right) V_{d d}^{2} f_{c l k}
$$

where $a$ is the switching activity of a repeater, which is assumed to be 0.15 , and $f_{c l k}$ is the clock frequency.

For the leakage power, we consider only the subthreshold leakage as in [2]. The subthreshold leakage current of a minimum-sized NMOS transistor is given by

$$
I_{o f f}=I_{o f f}^{r e f} \cdot 10^{\frac{\left(V_{t h}^{r e f}-V_{t h}\right)}{S_{w}}}
$$

where $I_{o f f}^{\text {ref }}$ and $V_{t h}^{\text {ref }}$ are some reference subthreshold leakage current and threshold voltage respectively for a technology, and $S_{w}$ is the subthreshold swing, which we assume $100 \mathrm{mV} /$ decade at the temperature $100^{\circ} \mathrm{C}$. The model assumes that the transistor is at OFF state when $V_{g s}=0$ and $V_{d s}=V_{d d}$. For the ease of calculation, we change the formula from base 10 to base $e$ and get

$$
I_{o f f}=I_{o f f}^{r e f} \cdot e^{\frac{\left(V_{t h}^{r e f}-V_{t h}\right)}{S_{w}^{\prime}}}
$$

where $S_{w}^{\prime}=\frac{S_{w}}{\log _{e} 10}$.
The average leakage power of a repeater is

$$
\begin{aligned}
P_{\text {leakage }} & =V_{d d} I_{\text {leakage }} \\
& =\frac{1}{2} V_{d d}\left(I_{o f f}^{n} W_{n_{m i n}}+I_{o f f}^{p} W_{p_{\min }}\right) s
\end{aligned}
$$

where $I_{o f f}^{n}$ and $I_{o f f}^{p}$ are the reference subthreshold leakage current for NMOS and PMOS transistors respectively, and $W_{m i n}^{n}$ and $W_{\text {min }}^{p}$ are the widths of the NMOS and PMOS transistors in a minimum-sized inverter.

The short circuit power dissipation depends on the transition time at the input and the output of an inverter. Assuming symmetric high-to-low and low-to-high transitions at the input and the output of the
repeater, the short circuit power is given by

$$
P_{\text {short-circuit }}=a t_{r} V_{d d} W_{\text {min }}^{n} s I_{\text {short-circuit }} f_{c l k}
$$

where $a$ is the same switching factor as in the dynamic power expression, and $t_{r}=\tau \log _{e} 3$.

The total power is given by

$$
P_{\text {repeater }}=P_{\text {dynamic }}+P_{\text {leakage }}+P_{\text {short-circuit }}
$$

Therefore, we have
$P_{\text {repeater }}=k_{1} V_{d d}^{2}\left(s\left(c_{p}+c_{o}\right)+l c\right)+k_{2} V_{d d} e^{\frac{\left(V_{t h}^{\text {ref }}-V_{t h}\right)}{S^{\prime}}} s+k_{3} V_{d d} s \tau$
where

$$
\begin{aligned}
k_{1} & =a f_{c l k} \\
k_{2} & =\frac{1}{2}\left(I_{o f f}^{n_{0}} W_{\min }^{n}+I_{o f f}^{p_{0}} W_{\min }^{p}\right) \\
k_{3} & =a W_{\min }^{n} f_{c l k} \log _{e} 3
\end{aligned}
$$

and

$$
\begin{equation*}
\frac{P_{\text {repeater }}}{l}=k_{1}\left(\frac{s}{l}\left(c_{p}+c_{o}\right)+c\right)+k_{2} \frac{s}{l}+k_{3} V_{d d} s \frac{\tau}{l} \tag{8}
\end{equation*}
$$

We specify the target delay by using $\left(\frac{\tau}{l}\right)_{o p t}(1+f)$, where f is the slack expressed in terms of the extra fraction of optimal unit length delay. By setting the net delay $\tau=(1+f)\left(\frac{\tau}{l}\right)_{\text {opt }} l$, we can simplify the expression by replacing $k_{3} \frac{\tau}{l}$ with $k_{3}^{\prime}=k_{3}(1+f)\left(\frac{\tau}{l}\right)_{o p t}$.

## 3. SINGLE NET POWER OPTIMIZATION

For an interconnect of length $L$, the total power dissipated by the inserted repeaters is $\frac{P_{\text {repeater }}}{l} L$, where $\frac{P_{\text {repeater }}}{l}$ is a function of $\left\{V_{d d}, V_{t h}, l, s\right\}$. Given a delay target specified in terms of $f$, the objective is to select from the feasible solutions the one which gives the minimum total power dissipation for the wire. Therefore, the problem can be formulated mathematically as

$$
\begin{array}{cl}
\min & \left(\frac{P_{\text {repeater }}}{l}\right)\left(V_{d d}, V_{t h}, l, s\right) \\
\text { subject to } & \left(\frac{\tau}{l}\right)\left(V_{d d}, V_{t h}, l, s\right)=(1+f)\left(\frac{\tau}{l}\right)_{o p t} \tag{9}
\end{array}
$$

In this section, we first review the method from [2], which solves Problem (9) with pre-defined $V_{d d}$ and $V_{t h}$ levels. Then we show that there exists a unique solution for Problem (9) and present a set of equations to solve the problem analytically. Finally we compare the results from power optimization with and without considering $V_{d d}$ and $V_{t h}$ as optimization variables.

### 3.1 Optimization under fixed Vdd and Vth

For given $V_{d d}, V_{t h}$, and a delay target, the optimal $l$ and $s$ that give the minimum $\frac{P_{\text {repeater }}}{l}$ can be obtained by solving the following set of nonlinear equations in [2], i.e.,

$$
\begin{align*}
\frac{\partial \frac{P_{\text {repeater }}}{l}}{\partial s} & =0  \tag{10}\\
\left(\frac{\tau}{l}\right)\left(V_{d d}, V_{t h}, l, s\right)-(1+f)\left(\frac{\tau}{l}\right)_{o p t} & =0 \tag{11}
\end{align*}
$$

The insertion length $l$ is a function of the repeater size $s$ under the equality delay constraint (11). In this problem, both the objective function and the constraint are posynomials. This type of problem is known to have a single local optimum that is also global, which can be obtained by setting the gradient of the objective function with respect to the design variables to zero.

### 3.2 Optimization with Vdd and Vth Tuning

When $V_{d d}$ and $V_{t h}$ are treated as variables, the functions are no longer posynomials. Therefore it is not clear whether there is only one local optimum. The new problem can be solved by an exhaustive search on $V_{d d}$ and $V_{t h}$ for the minimum power. For given $V_{d d}$ and $V_{t h}$, we obtain the minimum unit length power using the method in Section 3.1. Then we search on $V_{d d}$ and $V_{t h}$ for the minimum $\frac{P_{\text {repeater }}}{l}$. Figure 1 shows the resulting contour plot of $\frac{P_{\text {repeater }}}{l}$ versus $V_{d d}$ and $V_{t h}$. Each contour line represents the continuous combination of $V_{d d}$ and $V_{t h}$ that achieves the same value of $\frac{P_{\text {repeater }}}{l}$. The optimal value, which is a single point degenerated from a contour, locates right by the delay constraint line and is marked as $\left(V_{d d}^{o p t}, V_{t h}^{o p t}\right)$ in Figure 1. This plot shows


Figure 1: Contour plot of unit length power with Vdd and Vth as variables. The delay penalty is $5 \%$ of the optimal delay.
that there exists a unique optimum in the possible range of $V_{d d}$ and $V_{t h}$, which hints that the problem of power minimization through $V_{d d}$ and $V_{t h}$ can be solved analytically. Our future research will attempt to prove that this problem possesses a unique local optimum. On the other hand, based on the observation from the exhaustive search, we develop an efficient analytical method below to solve this problem. The analytical and exhaustive search methods obtain the same results in all our experiments.

In order to solve for the optimal point directly, we derive a set of nonlinear equations by setting the gradient of the objective function to zero. Following the equality delay constraint, one of the variable must be a function of the other three variables. In our derivation, $V_{t h}$ is chosen to be the dependent variable, because it is the only variable that can be easily expressed in the closed-form of the other three variables. From Equation (5), $V_{t h}$ can be expressed in terms of $V_{d d}$ and $r_{s}$ as

$$
V_{t h}=V_{d d}-\left(\frac{K_{3} V_{d d}}{r_{s}}\right)^{\frac{1}{\alpha}}
$$

By rearranging Equation (2), $r_{s}$ can be expressed as a function of $l$ and $s$ :

$$
r_{s}=\frac{(1+f)\left(\frac{\tau}{l}\right)_{o p t}-r s c_{o}-\frac{1}{2} r c l}{\frac{c_{o}+c_{p}}{l}+\frac{c}{s}}
$$

Therefore, when deriving the gradients of the objective function, $V_{t h}$ is treated as a function of $V_{d d}, l$ and $s$. The following equations set the gradients of the objective function with respect to $V_{d d}, s$ and
$l$ to zero.

$$
\begin{aligned}
& \frac{\partial \frac{P_{\text {repeater }}}{l}}{\partial V_{d d}}=2 k_{1} V_{d d}\left(\frac{s}{l}\left(c_{o}+c_{p}\right)+c\right) \\
&+k_{2} e^{\frac{-V_{t h}\left(V_{d d}, l, s\right)}{S_{w}^{\prime}}} \frac{s}{l} \\
&-\frac{1}{S^{\prime}} \frac{\partial V_{t h}}{\partial V_{d d}} k_{2} V_{d d} e^{\frac{-V_{t h}\left(V_{d d}, l, s\right)}{S_{w}^{\prime}}} \frac{s}{l}+k_{3}^{\prime} s=0 \\
&+\frac{\partial \frac{P_{\text {repeater }}}{l}}{\partial s} \\
&=k_{1} V_{d d}^{2} \frac{c_{o}+c_{p}}{l} \\
&-\frac{1}{S^{\prime}} \frac{\partial V_{t h}}{\partial s} e^{\frac{-V_{t h}\left(V_{d d}, l, s\right)}{S_{w}^{\prime}}} \frac{1}{l} \\
&-k_{d d} e^{\frac{-V_{t h}\left(V_{d d}, l, s\right)}{S_{w}^{\prime}}} \frac{s}{l}+k_{3}^{\prime} V_{d d}^{2}=0 \\
&\left.\frac{\partial \frac{P_{\text {repeater }}}{l}}{\partial l} c_{o}+c_{p}\right) \frac{s}{l^{2}} \\
&-\frac{1}{S^{\prime}} \frac{\partial V_{t h}}{\partial l} k_{2} V_{d d} e^{\frac{-V_{t h}\left(V_{d d}, l, s\right)}{S_{w}^{\prime}}} \frac{s}{l^{2}} \\
& S_{w} \\
& S_{w}^{\prime} \frac{s}{l}=0
\end{aligned}
$$

where

$$
\begin{aligned}
& \frac{\partial V_{t h}}{\partial V_{d d}}=1-\frac{1}{\alpha}\left(\frac{K_{3}}{r_{s}}\right)^{\frac{1}{\alpha}} V_{d d}^{\frac{1}{\alpha}-1} \\
& \frac{\partial V_{t h}}{\partial s}=\frac{1}{\alpha}\left(K_{3} V_{d d}\right)^{\frac{1}{\alpha}} r_{s}^{-\frac{1}{\alpha}-1} \frac{\partial r_{s}}{\partial s} \\
& \frac{\partial V_{t h}}{\partial l}=\frac{1}{\alpha}\left(K_{3} V_{d d}\right)^{\frac{1}{\alpha}} r_{s}^{-\frac{1}{\alpha}-1} \frac{\partial r_{s}}{\partial l}
\end{aligned}
$$

and

$$
\begin{aligned}
\frac{\partial r_{s}}{\partial s}= & \left(\frac{c_{o}+c_{p}}{l}+\frac{c}{s}\right)^{-2}\left\{\begin{array}{l}
\frac{c}{s^{2}}\left((1+f)\left(\frac{\tau}{l}\right)_{o p t}-\frac{1}{2} r c l\right) \\
-\frac{r c c_{o}}{s}-r c\left(\frac{c_{o}+c_{p}}{l}+\frac{c}{s}\right)
\end{array}\right\} \\
\frac{\partial r_{s}}{\partial l}= & -\frac{1}{2} r c\left(\frac{c_{o}+c_{p}}{l}+\frac{c}{s}\right)^{-1} \\
& +\left\{\frac{c_{o}+c_{p}}{l^{2}}\left((1+f)\left(\frac{\tau}{l}\right)_{o p t}-r s c_{o}-\frac{1}{2} r c l\right)\right\} \\
& \cdot\left(\frac{c_{o}+c_{p}}{l}+\frac{c}{s}\right)^{-2}
\end{aligned}
$$

These equations can be solved numerically using a standard nonlinear equation solver. We implement this in Matlab by using the command "fsolve".

### 3.3 Experimental Results

The methdology proposed is used to optimize unit length power for a single net. The parameters for the power and delay models across various technology nodes up to 65 nm are taken from [1]. Table 1 compares the proposed method to the method using fixed $V_{d d}$ and $V_{t h}$ in Section 3.1 respectively across different technology for target delay $\tau=(1+f)\left(\frac{\tau}{l}\right)_{o p t}$ where $f$ is between $5 \%$ and $100 \%$. The results from optimization under fixed $V_{d d}$ and $V_{t h}$ are called the reference values in this paper. The reference supply voltage $V_{d d}^{\text {ref }}$ used for each technology are obtained from [1] and $V_{t h}^{\text {ref }}$ values are assumed to be $25 \%$ of their respective $V_{d d}^{\text {ref }}$ as in [2].

As shown in Table 1, the amount of power saving that can be achieved from $V_{d d}$ and $V_{t h}$ optimization depends on the delay target. For $f=20 \%$, the power saving is up to $28 \%$ across all technology nodes. When $f=100 \%$, the power saving is more than $50 \%$ for

| node | f | $V_{d d}$ <br> $(\mathrm{~V})$ | $\frac{V_{\text {dd }}}{V_{d d}^{\text {ref }}}$ | $V_{t h}$ <br> $(\mathrm{~V})$ | $\frac{v_{\text {th }}}{V_{t h}^{\text {ref }}}$ | $s$ <br> $(\times \min )$ | $\frac{s}{s_{\text {ref }}}$ | $l$ <br> $(\mathrm{~mm})$ | $\frac{l}{l_{\text {ref }}}$ | $\left(\frac{P}{l}\right)_{\text {opt }}$ <br> $(\mathrm{W} / \mathrm{m})$ | $\left(\frac{P}{l}\right)_{\text {opt }}$ <br> saving |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 130 | $5 \%$ | 1.06 | 0.92 | 0.27 | 0.95 | 59.5 | 1.12 | 1.65 | 0.97 | 0.16 | $3 \%$ |
|  | $10 \%$ | 0.97 | 0.82 | 0.27 | 0.95 | 59.7 | 1.31 | 1.74 | 0.93 | 0.13 | $10 \%$ |
|  | $20 \%$ | 0.84 | 0.70 | 0.26 | 0.95 | 59.1 | 1.61 | 1.92 | 0.92 | 0.10 | $25 \%$ |
|  | $100 \%$ | 0.51 | 0.41 | 0.24 | 0.85 | 42.1 | 2.60 | 3.13 | 1.01 | 0.04 | $62 \%$ |
| 90 | $5 \%$ | 0.93 | 0.87 | 0.23 | 0.88 | 57.5 | 1.12 | 1.34 | 0.97 | 0.25 | $6 \%$ |
|  | $10 \%$ | 0.85 | 0.78 | 0.23 | 0.88 | 57.6 | 1.31 | 1.41 | 0.94 | 0.21 | $14 \%$ |
|  | $20 \%$ | 0.73 | 0.66 | 0.22 | 0.88 | 57.0 | 1.60 | 1.56 | 0.92 | 0.16 | $28 \%$ |
|  | $100 \%$ | 0.43 | 0.38 | 0.19 | 0.75 | 40.2 | 2.54 | 2.59 | 1.06 | 0.06 | $65 \%$ |
| 65 | $5 \%$ | 0.75 | 1.02 | 0.20 | 1.12 | 39.4 | 1.08 | 0.87 | 0.96 | 0.23 | $2 \%$ |
|  | $10 \%$ | 0.69 | 0.92 | 0.20 | 1.11 | 39.4 | 1.25 | 0.92 | 0.92 | 0.20 | $7 \%$ |
|  | $20 \%$ | 0.60 | 0.79 | 0.20 | 1.10 | 39.0 | 1.51 | 1.03 | 0.89 | 0.16 | $18 \%$ |
|  | $100 \%$ | 0.36 | 0.45 | 0.16 | 0.91 | 27.9 | 2.35 | 1.77 | 0.99 | 0.07 | $54 \%$ |

Table 1: Unit length power solutions from optimization with Vdd and Vth tunning and the comparison with optimization under fixed $V_{d d}$ and $V_{t h}$.
all generations. The power saving is mainly achieved by lowering the supply voltage. As we can see, the optimal $V_{d d}$ levels are generally lower than the reference values. The $V_{d d}$ level decreases with increasing slack $f$, showing that $V_{d d}$ provides good trade-off for power by utilizing $f$. The optimal $V_{t h}$ values slowly decreases with increasing $f$ to compensate for the loss of performance from $V_{d d}$ reduction. The reduction in $V_{t h}$ causes a moderate increase in leakage power, but is rewarded by a large decrease in the dynamic power from lowering $V_{d d}$. Repeater sizes $s$ are larger than the reference values to compensate for the loss of the drive strength due to $V_{d d}$ reduction. The segment lengths, $l$, stay relatively close to the reference values in all cases.

## 4. FULL-CHIP INTERCONNECT POWER

In this section, we propose a methodology to evaluate full-chip interconnect power. In [7], a closed-form analytical expression of the wire-length distribution for on-chip random logic networks based on Rent's rule is developed. We estimate the full-chip power by integrating the unit length power over the wire-length distribution from the smallest wire length with non-negligible power to the longest global interconnect assumed by the wire-length distribution model. We use the delay optimal segment length $l_{o p t}$ given by Equation (6) to define the shortest interconnect that requires repeater insertion. Nets shorter than $l_{o p t}$ are not considered as they do not need repeaters. The delay of each net is bounded by $90 \%$ of the clock period $T_{c l k}$ as in [8]. For an interconnect of length $L$ operating at $V_{d d}$ and $V_{t h}$, the optimal delay is

$$
D_{o p t}=\left(\frac{\tau}{l}\right)_{o p t}\left(V_{d d}, V_{t h}\right) L
$$

where $\left(\frac{\tau}{l}\right)_{o p t}\left(V_{d d}, V_{t h}\right)$ is given by Equations (5) and (7). The difference between $D_{o p t}$ and $0.9 \cdot T_{\text {clk }}$ is the slack that we can use to optimize its power. We define $L_{\max }$ to be the longest interconnect length which satisfies the target delay with delay optimal repeater insertion, i.e.,

$$
L_{\max }=\frac{0.9 \cdot T_{c l k}}{\left(\frac{\tau}{l}\right)_{o p t}}
$$

We pipeline the interconnects of lengths larger than $L_{\max }$ so that the length of each segment is smaller than $L_{\max }$. We assume that the delay overhead of pipelining flip-flops is amortized in $0.1 \cdot T_{\text {clk }}$. Therefore, the power for the full-chip is given by

$$
\begin{equation*}
P=\int_{\nu_{o p t}}^{2 \sqrt{N}} \mathbf{R}(\nu)\left(\frac{P}{l}\right)_{o p t}(f) l_{\beta} \beta d \nu \tag{12}
\end{equation*}
$$

where

| $\nu$ | wire length in terms of gate pitches; |
| :--- | :--- |
| $\nu_{o p t}$ | $l_{o p t}$ in terms of gate pitches; |
| $N$ | number of logic gates; |
| $\beta$ | number of pipelining stages; |
| $l_{\beta}$ | wire length per stage; <br> $\mathbf{R}(\nu)$ <br> $\left(\frac{P}{l}\right)_{o p t}(f)$ <br> wirelength distribution function; <br> power per length function defined in the <br> $f$ |
| Problem Formulation (9); <br> slack in terms of multiple of $\left(\frac{\tau}{l}\right) ;$ |  |

The length in terms of gate pitches is obtained by

$$
\begin{equation*}
\nu=\frac{l}{\sqrt{A F} \mathbf{T}} \tag{13}
\end{equation*}
$$

where $A F$ is the gate area factor, which is 320 across all technology nodes and $\mathbf{T}$ is the technology node in terms of minimum local metal's half-pitch dimension. The number of pipelining stages $\beta$ and the wire length per stage $l_{\beta}$ are given by

$$
\begin{aligned}
\beta & =\left\lceil\frac{\nu \sqrt{A F} \mathbf{T}}{L_{\max }}\right\rceil, \\
l_{\beta} & =\frac{\nu \sqrt{A F} \mathbf{T}}{\beta}
\end{aligned}
$$

The optimal power per length $\left(\frac{P}{l}\right)_{\text {opt }}$ is a function of the target delay, and is obtained using the method discussed in Section 3.1 when $V_{d d}$ and $V_{t h}$ are fixed and that in Section 3.2 when $V_{d d}$ and $V_{t h}$ are design variables. Target delay of an interconnect of length $l_{\beta}$ is again specified by $\tau=(1+f)\left(\frac{\tau}{l}\right)_{o p t}\left(V_{d d}, V_{t h}\right) l_{\beta}$, where

$$
f=\frac{0.9 \cdot T_{c l k}}{\left(\frac{\tau}{l}\right)_{o p t} \cdot l_{\beta}}-1
$$

| Technology Node (nm) | 130 | 90 | 65 | 45 |
| :--- | :---: | :---: | :---: | :---: |
| $\#$ transistors (M) | 97 | 193 | 276 | 1546 |
| $T_{c l k}(\mathrm{ps})$ | 594 | 251 | 148 | 86.9 |
| $V_{d d}(\mathrm{~V})$ | 1.1 | 1 | 0.7 | 0.6 |
| $V_{t h}(\mathrm{~V})$ | 0.28 | 0.25 | 0.17 | 0.15 |
| $L_{\max }(\mathrm{mm})$ | 6.94 | 2.30 | 1.06 | 0.513 |
| $l_{\text {opt }}(\mathrm{mm})$ | 1.32 | 1.06 | 0.67 | 0.540 |

Table 2: List of parameters based on 2001 ITRS.
Note: The number of gates $N$ is assumed to be \# transistors/4

### 4.1 Vdd and Vth Optimization

To optimize the full-chip interconnect power, we consider various cases of $V_{d d}$ and $V_{t h}$ assignment for nets. Practical assignment has
limited number of $V_{d d}$ and $V_{t h}$ levels throughout the chip. Multiple $V_{d d}$ levels are provided either by having multiple power distribution networks or by inserting pass transistors to create lower $V_{d d}$ supplies than the system $V_{d d}$. Multiple $V_{t h}$ can be achieved either through selective transistor doping or through substrate biasing. The $V_{d d}$ and $V_{t h}$ pair for a net can be formed from any one of the available $V_{d d}$ and $V_{t h}$ levels. Therefore, increasing $V_{d d}$ and $V_{t h}$ levels improves the power saving it can achieve due to more fine-grained control to $V_{d d}$ and $V_{t h}$ for each net. We are interested in maximizing the power saving that can be achieved by the minimum number of $V_{d d}$ and $V_{t h}$ levels available at the full-chip level, since extra $V_{d d}$ and $V_{t h}$ levels increase area and manufacturing costs. We compare the optimal full-chip global interconnect power of each combination ( $N_{d d}, N_{t h}$ ), where $N_{d d}$ is the number of $V_{d d}$ levels and $N_{t h}$ is the number of $V_{t h}$ levels. The theoretical optimum power occurs at $N_{d d} \rightarrow \infty$ and $N_{t h} \rightarrow \infty$, i.e., the $V_{d d}$ and $V_{t h}$ of each net can be taylored. Such comparison provides us with an idea of the potential power saving by increasing $N_{d d}$ and $N_{t h}$.

Table 3 shows our searching algorithm for the power optimal $V_{d d}$ and $V_{t h}$ levels at the full-chip level. Given $N_{d d}$ and $N_{t h}$, the algorithm first generates all possible combinations of $V_{d d}$ and $V_{t h}$ for the full-chip at line 3. For a particular $N_{d d}$ levels of $V_{d d}$ and $N_{t h}$ levels of $V_{t h}$, any combination of $\left(V_{d d}, V_{t h}\right)$ that has lower delay per length than the reference combination $\left(V_{d d}^{\text {ref }}, V_{t h}^{\text {ref }}\right)$, which provides the best delay performance, is discarded. Combinations which cannot even achieve the delay bound at the shortest wire length $l_{\text {opt }}\left(V_{d d}^{\text {ref }}, V_{t h}^{\text {ref }}\right)$ in our defined global interconnect are also discarded. These are implemented in line 5. The algorithm then evaluates $L_{\max }\left(V_{d d}, V_{t h}\right)$, which is the maximum wire length that satisfies the $0.9 \cdot T_{c l k}$ delay bound, for every $\left(V_{d d}, V_{t h}\right)$ combination. The combinations are then sorted as in line 6 , such that the nets of different lengths are assigned with $V_{d d}$ and $V_{t h}$ as illustrated in Figure 2. Finally, the power of each of these regions with different $\left(V_{d d}, V_{t h}\right)$ assignments are computed in lines 9-14. Note that wires of length larger than $L_{\max }\left(V_{d d}^{\text {ref }}, V_{t h}^{\text {ref }}\right)$ have to be broken down into segments by means of pipelining as discussed, which is implemented by looping on the number of pipeline stages at line 10 and by folding the integration bounds in lines $11-12$. $\nu$ is simply the length in terms of gate pitches, and the conversion between $\nu$ and length in absolute dimensions are done using Equation (13). Also note that the optimal power per length function $\left(\frac{P}{l}\right)\left(f, V_{d d}, V_{t h}\right)_{o p t}$ in line 13 refers to the power optimal repeater insertion with fixed $V_{d d}$ and $V_{t h}$ discussed in Section 3.1.


Figure 2: $\left(V_{d d}, V_{t h}\right)$ assignment in a net distribution

The ideal case in which $N_{d d} \rightarrow \infty$ and $N_{t h} \rightarrow \infty$ can be computed by the same algorithm with some modification. Even though some smart pruning has been done to the search space as shown in Table 3, the algorithm fundamentally performs exhaustive

| Algo | rithm: ComputeOptPower ( $N_{d d}, N_{\text {th }}$ ) |
| :---: | :---: |
| 1. $S\left(V_{d d}\right)=$ the set of $V_{d d}$ levels to search |  |
| 2. $S\left(V_{t h}\right)=$ the set of $V_{t h}$ levels to search |  |
| 3. $S\left(\left\{V_{d d}\right\},\left\{V_{t h}\right\}\right)$ |  |
| 4. for each $\{V$ |  |
| $\begin{array}{ll} 5 . & \text { remove combinations }\left(V_{d d}, V_{t h}\right) \in\left\{V_{V d}\right\} \times\left\{V_{t h}\right\} \\ \text { s.t. } & L_{\max }\left(V_{d d}, V_{t h}\right)<l_{\text {opt }}\left(V_{d d}^{\text {ref }}, V_{t h}^{\text {ref }}\right) \text { or } \\ & \left(\frac{\tau}{l}\right)_{\text {opt }}\left(V_{d d}, V_{t h}\right)>\left(\frac{\tau}{l}\right)_{\text {opt }}\left(V_{d d}^{\text {ref }}, V_{t h}^{\text {ref }}\right) \end{array}$ |  |
| 6. $\mathcal{S}=\operatorname{sorted}\left(V_{d d}, V_{t h}\right)$ combinations in the ascending order of $L_{\max }\left(V_{d d}, V_{t h}\right)$ |  |
| 7. $\mathrm{P}=0$ |  |
| 8. $\mathrm{LB}=\nu_{d}^{o p t}$ |  |
| 9. for each $\left\{V_{d d}, V_{t h}\right\} \in \mathcal{S}$ |  |
| 10. for $\mathrm{p}=0$ to $\beta-1$ |  |
| 11. $\quad \mathrm{T}=\min \left(2 \sqrt{N},(p+1) \nu_{\max }\left(V_{d d}, V_{t h}\right)\right)$ |  |
| 12. $\perp=\max \left((p+1) \mathrm{LB},(p+1) \nu_{\max }\left(V_{d d}, V_{t h}\right)\right.$ |  |
| 13. $\mathrm{P}+=\int_{\perp}^{\top} \mathbf{R}(\nu)\left(\frac{P}{l}\right)_{o p t}\left(f, V_{d d}, V_{t h}\right) l_{\beta} \beta$ |  |
|  | $\mathrm{LB}=\nu_{\max }\left(V_{d d}, V_{t h}\right)$ |
|  | mark the set $\left\{V_{d d}\right\},\left\{V_{t h}\right\}$ as optimal if $P$ is the minimum power found |

Table 3: Optimal $V_{d d}$ and $V_{t h}$ levels search
search, in which the number of combinations for $\left(V_{d d}, V_{t h}\right)$ grows exponentially as $N_{d d}$ and $N_{t h}$ increase. We have found that $N_{d d}$ and $N_{t h}$ beyond 3 is impractical from the runtime perspective. Therefore, instead of using large $N_{d d}$ and $N_{t h}$, the power per length function is changed to one which makes use of our $\left(\frac{P}{l}\right)_{o p t}(f)$ function in Section 3.2, and $N_{d d}=N_{t h}=1$. This is equivalent to finding the optimum repeater insertion with computed optimum $V_{d d}$ and $V_{t h}$ for each net.

### 4.2 Experimental Results

The methodology discussed above is used to optimize the fullchip power of chip sizes reported in [1] for various technology generations. $N_{d d}$ and $N_{t h}$ are enumerated only up to three for the sake of runtime. $V_{d d}$ and $V_{t h}$ search range are minimized without compromising the power optimality. Figure 3 shows the full-chip power of various $V_{d d}$ and $V_{t h}$ configurations, where each pair on the x-axis is ( $N_{d d}, N_{t h}$ ). The highest performance (the most power consuming) combination $\left(V_{d d}^{r e f}, V_{t h}^{r e f}\right)$ is always retained in all configurations by default, therefore the configuration $(1,1)$ refers to the optimal full-chip power with fixed reference $V_{d d}$ and $V_{t h}$ for all nets. The "ideal" combination refers to the continuous $V_{d d}$ and $V_{t h}$ assignment, i.e., $N_{d d}, N_{t h} \rightarrow \infty$. Power reduces by $47 \%, 28 \%$ and $13 \%$ for $130 \mathrm{~nm}, 90 \mathrm{~nm}$ and 65 nm technology nodes respectively by going from the single $V_{d d}$, single $V_{t h}$ configuration to the dual $V_{d d}$, dual $V_{t h}$ configuration. Using dual $V_{t h}$ instead of single $V_{t h}$ under dual $V_{d d}$ only gives $\sim 3 \%$ power reduction, as opposed to the $20 \%$ plus reduction reported for logic circuits in [5]. This suggests that optimizing the single reference $V_{t h}$ may just perform as well as the dual $V_{t h}$ configuration in terms of power consumption. The dual $V_{d d}$ and dual $V_{t h}$ configuration has the total power just $17 \%$, $12 \%$ and $5 \%$ from the theoretical power optimum configuration which allows infinite $V_{d d}$ and $V_{t h}$ levels. Moreover, we observe no significant improvement by moving to combinations with more $V_{d d}$ and $V_{t h}$ levels in all technology generations.

The power breakdown of the optimized full-chip interconnect for each $\left(N_{d d}, N_{t h}\right)$ configuration is shown in each bar in Figure 3. Multiple $V_{d d}$ configurations (i.e., $N_{d d}>1$ ) in 130 nm and 90 nm technology nodes achieve significant dynamic power saving


Figure 3: Power of optimized nets under different $N_{d d}$ and $N_{t h}$. Each group of bars contain results for $130 \mathrm{~nm}, 90 \mathrm{~nm}$ and 65 nm technology nodes.
by aggressively reducing the second $V_{d d}$ level, as shown in Table 4. The threshold voltage of the second $V_{t h}$ level slightly decreases to compensate for the loss of performance due to $V_{d d}$ reduction, at the expense of slight increase in the leakage power. On the other hand, the leakage power in 65 nm technology node is comparatively a lot larger in the $(1,1)$ configuration. From Table 4, the second $V_{t h}=0.2 \mathrm{~V}$ leaps above the reference level of 0.175 V to limit the growth of leakage power. This can be seen in Figure 3, where the block of leakage for the 65 nm bars slightly reduces from $(1,1)$ to multi- $V_{d d} / V_{t h}$ configurations. Therefore, we conclude that in order to get the right balance between dynamic power and leakage power for total power reduction in interconnect, we must consider both $V_{d d}$ and $V_{t h}$ optimization.

| Tech Node <br> $(\mathrm{nm})$ | $\left(N_{d d}, N_{t h}\right)$ | $V_{d d} \mathrm{~s}$ <br> $(\mathrm{~V})$ | $V_{t h} \mathrm{~s}$ <br> $(\mathrm{~V})$ |
| :---: | :---: | :---: | :---: |
| 130 | $(2,1)$ | $1.1,0.572$ | 0.275 |
|  | $(2,2)$ | $1.1,0.506$ | $0.226,0.275$ |
| 90 | $(2,1)$ | $1,0.64$ | 0.25 |
|  | $(2,2)$ | $1,0.64$ | $0.2,0.25$ |
| 65 | $(2,1)$ | $0.7,0.532$ | 0.175 |
|  | $(2,2)$ | $0.7,0.532$ | $0.175,0.2$ |

Table 4: $V_{d d}$ and $V_{t h}$ levels for each $\left(N_{d d}, N_{t h}\right)$
Figure 4 shows the breakdown of total wire length being assigned to ( $V_{d d}, V_{t h}$ ) marked on each region of the figure for the dual $V_{d d}$, dual $V_{t h}$ case. The regions are ordered in the increasing power (the decreasing delay) ( $V_{d d}, V_{t h}$ ) combinations from the bottom to the top. A large portion of the net is assigned to the combination which has $V_{t h} / V_{d d}$ ratio way above the default 0.25 , particularly for 65 nm technology. This implies that the $V_{t h} / V_{d d}$ ratio has to be increased in order to attain power optimality. This is in line with the conclusion made by other works in the literature [9], which suggests that the $V_{t h} / V_{d d}$ ratio has to be larger than that current designs use.

## 5. CONCLUSIONS

This paper studies the opportunity of power saving by computing power optimal repeater sizes, repeater insertion lengths, and for the first time $V_{d d}$ and $V_{t h}$ levels for both single nets and a full chip. We have derived a set of analytical formulae which finds the optimal interconnect power given the amount of the timing slack on a single


Figure 4: Net length distribution for dual Vdd, dual Vth configuration
net. Compared to [2] which does not consider $V_{d d}$ and $V_{t h}$ as design variables, our method that customizes $V_{d d}$ and $V_{t h}$ for each net can reduce power by more than $50 \%$ for both single nets and at the chip level. We have also studied the power saving of using multiple $V_{d d}$ and $V_{t h}$ levels for buffering interconnects. Power reduces by $47 \%, 28 \%$ and $13 \%$ for $130 \mathrm{~nm}, 90 \mathrm{~nm}$ and 65 nm technology nodes respectively by going from the single $V_{d d}$, single $V_{t h}$ configuration to the dual $V_{d d}$, dual $V_{t h}$ configuration. The fact that majority of the nets favors a $V_{d d}$ to $V_{t h}$ ratio of more than 0.35 across all generations suggests that the ratio of 0.25 as suggested by other works in the literature is too low for power optimality. We show that the dual $V_{d d}$ and dual $V_{t h}$ configuration is within $17 \%, 12 \%$ and $5 \%$ of the theoretical optimal power computed from our analytical method for $130 \mathrm{~nm}, 90 \mathrm{~nm}$ and 65 nm technology node; and that extra $V_{d d}$ or $V_{t h}$ level beyond dual $V_{d d}$ and dual $V_{t h}$ only gives marginal improvement. Our experiment also shows that multiple $V_{t h}$ does not improve power of interconnect as much as that of logic circuits.

Our future work focuses on evaluating the suggested systemwide $V_{d d}$ and $V_{t h}$ for power optimality of both logic circuits and interconnects. One assumption in this work is that we treat the reference combination $\left(V_{d d}^{r e f}, V_{t h}^{r e f}\right)$ as always available for nets' selection, while other combinations of $V_{d d}$ and $V_{t h}$ are explored. This assumption is reasonable since we assume all other parts of the chip have at least the reference supply and threshold voltage used by logic circuits. However, we may achieve better overall power by re-designing the system power supply and threshold voltages. In the future we will remove such restriction and allow system-wide $V_{d d}$ and $V_{t h}$ exploration.

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