Optimal Wiresizing for Interconnects with Multiple Sources

JASON CONG and LEI HE
University of California, Los Angeles

In this paper, we study the optimal wiresizing problem for nets with multiple sources under the RC tree model and the Elmore delay model. We decompose the routing tree for a multisource net into the source subtree (SST) and a set of loading subtrees (LSTs), and show that the optimal wiresizing solution satisfies a number of interesting properties, including: the LST separability, the LST monotone property, the SST local monotone property, and the dominance property. Furthermore, we study the optimal wiresizing problem using a variable segment-division rather than an a priori fixed segment-division as in all previous works and reveal the bundled refinement property. These properties lead to efficient algorithms to compute the optimal solutions. We have tested our algorithm on nets extracted from the multilayer layout for a high-performance Intel microprocessor. Accurate SPICE simulation shows that our methods reduce the average delay by up to 23.5% and the maximum delay by up to 37.8%, respectively, for the submicron CMOS technology when compared to the minimal wire width solution. In addition, the algorithm based on the variable segment-division yields a speedup of over 100× time and does not lose any accuracy, when compared with the algorithm based on the a priori fixed segment-division.

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1. INTRODUCTION

Interconnect delay has become the dominating factor in determining system performance in deep submicron VLSI designs. Recently developed techniques for interconnect delay minimization in the physical design level

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Authors' addresses: Department of Computer Science, University of California, Los Angeles, CA 90095; email: ⟨cong@cs.ucla.edu⟩; ⟨helei@cs.ucla.edu⟩.
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fall into two categories. One is topology optimization, such as the constructions of bounded-radius bounded-cost trees [Cong et al. 1992], AHHK trees [Alpert et al. 1993], A-trees [Cong et al. 1993], low-delay trees [Boese et al. 1993], and IDW/CFD trees [Hong et al. 1993]. In essence, these methods construct an interconnect tree to minimize both the total tree length and the paths between the input pin (also called the source) and a set of timing-critical output pins (also called critical sinks), whereas the conventional Steiner tree algorithms minimize only the total tree length. In addition, the nontree routing for delay minimization was explored in McCoy and Robins [1994] and Xue and Kuh [1995].

The other type of interconnect optimization methods is wiresizing optimization, which computes optimal wire width for each wire segment in the interconnect to minimize the interconnect delay. In the earlier works [Cong et al. 1993; Cong and Leung 1993, 1995], Cong et al. modeled the routing tree as a distributed RC tree and formulated the wiresizing problem under the Elmore delay model to minimize a weighted average delay from the source to a set of critical sinks. They developed the first polynomial-time optimal algorithm based on the separability, the monotone property, and the dominance property. Later on, Sapatnekar [1994] used the sensitivity-based heuristic and the convex programming technique [Sancheti and Sapatnekar 1994] for the optimal wiresizing problem to minimize the maximum interconnect delay, since the Elmore delay along a RC tree is a posynomial function of wire widths as first pointed out in Fishburn and Dunlop [1985]. Other recent wiresizing works include the greedy algorithm for performing wiresizing during tree construction [Hodes et al. 1994], the nonuniform wiresizing during the multilink insertion for existing interconnects [Xue and Kuh 1995], and the continuous nonuniform wiresizing based on a Lagrangian relaxation procedure [Chen et al. 1996a,b]. The Elmore delay model is used in Hodes et al. [1994], Xue and Kuh [1995], Chen et al. [1995a,b]. In addition, more accurate delay models are used in the following works: the moment-fitting based wiresizing [Menezes et al. 1994] under a higher-order RC delay model, and the sensitivity-based wiresizing [Xue et al. 1996] under a lossy transmission line model for trees of transmission lines.

Recently, wiresizing has been conducted simultaneously with device sizing to further reduce the interconnect delay. Cong and Koh [1994] formulated the simultaneous driver and wire sizing (SDWS) problem under the Elmore delay model. They revealed a dominance relation between the driver sizing and the correspondent optimal wire sizing, and developed an efficient SDWS algorithm. Later, Menezes et al. [1995] treated the simultaneous gate and wire-sizing problem as a posynomial program under both the Elmore delay model and a higher-order RC delay model. The posynomial program is further transformed into a convex program and then solved by the sequential quadratic programming method. Furthermore, Lillis et al. [1995] proposed a simultaneous wiresizing and buffer insertion algorithm under the Elmore delay model, based on the dynamic programming

scheme originally developed for the optimal buffer placement of van Ginneken [1990].

All these interconnect optimization methods assume that there is a unique source in each interconnect tree (called a *single source interconnect tree* (*SSIT*)) and minimize the delay between the source and a set of critical sinks. However, there exist many important interconnect structures with multiple potential sources, each driving the interconnect at a different time, such as those in global signal buses. None of the existing interconnect optimization methods consider such multisource interconnect trees (MSITs), except a very recent work by Cong and Madden [1995], where an MSIT topology optimization method based on the construction of min-cost min-diameter A-trees was developed, and a smaller delay was achieved when compared with the conventional routing methods. Although those single-source wiresizing algorithms based on mathematical programming[1] or sensitivity analysis [Sapatnekar 1994; Xue et al. 1996] might be adapted to minimize the delay between the multiple source-sink pairs by modifying their objective functions, it is of both theoretical and practical interest to understand the properties of the optimal wiresizing solutions for MSITs and develop efficient algorithms directly for MSITs.

In this paper, we study the optimal wiresizing problem for MSITs under the RC tree model and the Elmore delay model. We decompose an MSIT into a source subtree (SST) and a set of loading subtrees (LSTs), and show a number of interesting properties of the optimal wiresizing solutions under this decomposition, including: the LST separability, the LST monotone property, the SST local monotone property, and the dominance property. These properties lead to effective algorithms to compute the optimal wire width assignment for any given MSIT. We have tested our algorithm on multisource nets extracted from the multilayer layout of a high-performance Intel processor. SPICE simulation shows that our methods reduce the average delay by up to 23.5% and the maximum delay by up to 37.8% for the submicron CMOS technology. Furthermore, we study the optimal wiresizing problem using a variable segment-division rather than an a priori fixed segment-division as used in all previous works. We show the bundled refinement property that leads to a very efficient wiresizing algorithm based on bundled refinement operations and segment-division refinement operations. The algorithm yields a speedup of over 100× time and does not lose any accuracy, when compared to the method based on a priori fixed segment-division. Finally, we also investigate the fidelity of the Elmore delay model for wiresizing optimization using the ranking technique similar to that of Boese et al. [1993]. We have found that the optimal wiresizing solution selected according to the Elmore delay model is about 0.06% worse than the optimal wiresizing solution selected according to the SPICE-computed delay, when the delays of both solutions are measured by SPICE simulation. This experiment convincingly justifies our formulation

[1]Sancheti and Sapatnekar [1994], Menezes et al. [1994, 1995], Xue and Kuh [1995], Chen et al. [1996a].

based on the Elmore delay model for the current submicron CMOS technology. To the best of our knowledge, it is the first work that presents an in-depth study of both the optimal wiresizing problem for MSITs and the optimal wiresizing problem under a variable segment-division.

The remainder of this paper is organized as follows. In Section 2, we present the formulation of the MSIT wiresizing problem. In Sections 3 and 4, we study the properties of the optimal wiresizing solutions for MSIT designs, under the a priori fixed and the variable segment-divisions, respectively. These properties lead to efficient algorithms given in Section 5. Section 6 shows experimental results, including the fidelity study of the Elmore delay model. Section 7 concludes the paper with discussions of future work. The proofs of the Theorems 3, 5, and 6 are given in the Appendix. Proofs of other theorems, together with more experimental results, can be found in a technical report [Cong and He 1995b].

2. PROBLEM FORMULATION

2.1 Multisource Wiresizing (MSWS) Problem

We call the wiresizing problem for MSITs the *multisource wiresizing* (*MSWS*) *problem*. For an MSIT, each pin in the MSIT can be a source (driver), or a sink (receiver), or both at different times. We assume, however, no two sources in the MSIT are active at the same time. Let a *node* be either a pin or a Steiner point in the MSIT and $src(MSIT)$ the set of pins that can be sources of the MSIT. We say that $sink^i(MSIT)$ is the set of sinks in the MSIT when pin N_i is the source of the MSIT. In addition, let a *segment* connect two nodes and $\{S_1, S_2, \ldots, S_m\}$ be the set of segments in the MSIT. In order to capture the distributed resistive property of interconnects and achieve better wiresizing solutions, a segment is divided into a sequence of *unisegments*. The term "unisegment" is coined based on this assumption that the wire width is uniform within a unisegment. The *segment-division* determines the set of all unisegments, $\{E_1, E_2, \ldots, E_n\}$, in the MSIT. Our wiresizing problem is formulated to find a wire width for each unisegment from a set of given choices $\{W_1, W_2, \ldots, W_r\}$ $(W_1 < W_2 < \cdots < W_r)$. Different from our formulation, a segment in Cong and Leung [1993, 1995] is not further divided and is simply treated as a unisegment;[2] a segment in Cong and Koh [1994] is divided into a sequence of wires of unit length, and such a wire of unit length is treated as a unisegment. Thus, both segment-divisions in Cong and Leung [1993, 1995] and Cong and Koh [1994] are given a priori and fixed during the wiresizing procedure. In our formulation, the segment-division is in fact a variable during the wiresizing procedure and is defined by the wiresizing procedure, which is discussed later in Section 4. For simplicity, we assume that an a priori fixed segment-division is given in this section.

[2]We note that artificial degree-2 Steiner points can be introduced within a segment in Cong and Leung [1993, 1995] to achieve certain segment-division.

The modeling technique similar to those used in Cong and Leung [1993, 1995] and Cong and Koh [1994] is applied. Each unisegment is treated as a π-type RC circuit containing resistance r_E and capacitance c_E, respectively. Let the unit-width unit-length wire have wire resistance r_0, wire area capacitance c_0, and wire fringing capacitance c_1; then $r_E = r_0 \cdot (l_E/w_E)$ and $c_E = c_0 \cdot w_E \cdot l_E + c_1 \cdot l_E$ for unisegment E with width w_E and length l_E. The driver at source N_i is modeled by an output capacitance C_d^i and a fixed-value resistor R_d^i connected to an idle voltage source, and the receiver at sink N_j by a loading capacitor c_s^j. Thus, a given interconnect including its drivers and receivers is modeled by a distributed RC tree. The Elmore [1948] delay t^{ij} in the RC tree from source N_i to sink N_j is a function of the segment-division $\mathscr{E}$ and the wiresizing solution $\mathscr{W}$. It can be written as Equation (1) according to the Elmore delay formulation for RC trees [Rubinstein et al. 1983].

$$t^{ij}(\text{MSIT}, \mathscr{E}, \mathscr{W}) = \sum_{E \in P(N_i, N_j)} r_E \cdot \left(\frac{c_E}{2} + C_E \right), \tag{1}$$

where the summation is taken over all unisegments on the unique path $P(N_i, N_j)$ from source N_i to sink N_j, and C_E is the total downstream capacitance of unisegment E with respect to source N_i. In order to handle multiple source-sink pairs, we further introduce the following weighted delay formulation Equation (2).

$$t(\text{MSIT}, \mathscr{E}, \mathscr{W}) = \sum_{N_i \in \text{src(MSIT)}} \sum_{N_j \in \text{sink}^i\text{(MSIT)}} \lambda^{ij} \cdot t^{ij}(\text{MSIT}, \mathscr{E}, \mathscr{W}), \tag{2}$$

where λ^{ij} is the penalty weight to indicate the priority of the Elmore delay t^{ij} between source N_i and sink N_j.

With these definitions, we give the general formulation of the MSWS problem as follows.

Formulation 1. Given an MSIT, a segment-division $\mathscr{E}$ and a set of possible wire width choices, the multisource wiresizing (MSWS) problem for delay minimization is to determine a wiresizing solution $\mathscr{W}$ that gives a wire width w_E for every unisegment E under $\mathscr{E}$, such that the weighted delay $t(\text{MSIT}, \mathscr{E}, \mathscr{W})$ is minimized.

When there is only one source in an interconnect tree, the MSWS problem degenerates into the single-source wiresizing (SSWS) problem. Note that we assume a given segment-division in Formulation 1. A more general wiresizing problem, the multisource wiresizing problem without an a priori given segment-division (the *MSWS/E* problem) is presented in Section 4.

2.2 Weighted Delay Formulation

For simplicity, we assume that all interconnects belong to the same layer and the assumption is removed later in Section 3.3. It is not difficult to

verify that the Elmore delay t^{ij} between source N^i and sink N^j can be formulated as follows.

$$t^{ij}(\text{MSIT}, \mathscr{E}, \mathscr{W}) = \mathscr{K}_0^{ij} + \mathscr{K}_1^i \cdot \sum_{E \in \text{MSIT}} l_E \cdot w_E + \mathscr{K}_2 \cdot \sum_{E,E' \in \text{MSIT}} f^{ij}(E, E') \cdot \frac{l_E \cdot l_E \cdot w_{E'}}{w_E} + \mathscr{K}_3 \cdot \sum_{E,E' \in \text{MSIT}} f^{ij}(E, E') \cdot \frac{l_E \cdot l_{E'}}{w_E} + \mathscr{K}_4 \cdot \sum_{E} g^{ij}(E) \cdot \frac{l_E}{w_E} + \mathscr{K}_5 \cdot \sum_{E \in \text{MSIT}} h^{ij}(E) \cdot \frac{l_E^2}{w_E}, \tag{3}$$

where w_E and l_E are, respectively, the (wire) width and length of the unisegment E. $\mathscr{K}_0^{ij}$, $\mathscr{K}_1^i$, $\mathscr{K}_2$, . . . , $\mathscr{K}_5$ are constants independent of the wiresizing solution, as given in the following.

$$\mathscr{K}_0^{ij} = R_d^i \cdot C_d^i + R_d^i \cdot \sum_{u \in \text{sink}^i(\text{MSIT})} c_s^u + R_d^i \cdot \sum_{E \in \text{MSIT}} c_1 + \sum_{E \in P(N_i, N_j)} \frac{r_0 \cdot c_0}{2}$$

$$\mathscr{K}_1^i = R_d^i \cdot c_0$$

$$\mathscr{K}_2 = r_0 \cdot c_0$$

$$\mathscr{K}_3 = r_0 \cdot c_1$$

$$\mathscr{K}_4 = r_0$$

$$\mathscr{K}_5 = \frac{r_0 \cdot c_1}{2}.$$

Recall that R_d^i and C_d^i are the driving resistance and output capacitance for the driver at source N_i, and c_s^u the sink capacitance at sink N_u. These parameters can take account the different sizes of drivers/receivers at different sources/sinks of an MSIT. Besides, $f^{ij}(E, E')$, $g^{ij}(E)$, and $H^{ij}(E)$ defined in the following, again are constants independent of the wiresizing solution.

$$f^{ij}(E, E') = \begin{cases} 1 & \text{if } E \in P(N_i, N_j) \text{ and } E' \in \text{Des}^i(E) \\ 0 & \text{otherwise,} \end{cases} \tag{4}$$

$$g^{ij}(E) = \begin{cases} \sum_{u \in \text{sink}^i(E)} c_s^u & \text{if } E \in P(N_i, N_j) \\ 0 & \text{otherwise,} \end{cases} \tag{5}$$

$$h^{ij}(E) = \begin{cases} 1 & \text{if } E \in P(N_i, N_j) \\ 0 & \text{otherwise,} \end{cases} \tag{6}$$

where $\text{Des}^i(E)$ is the set of downstream unisegments of E with respect to source N_i, and $\text{sink}^i(E)$ the set of downstream sinks of E with respect to source N_i.

Assume that λ^{ij}s are normalized, that is,

$$\sum_{N_i \in src(\text{MSIT})} \sum_{N_j \in \text{sink}^i(\text{MSIT})} \lambda^{ij} = 1,$$

the objective function Equation (2) becomes:

$$\begin{aligned} t(\text{MSIT}, \mathscr{C}, \mathscr{W}) = {} & \mathscr{K}_0 + \mathscr{K}_1 \cdot \sum_{E \in \text{MSIT}} l_E \cdot w_E \\ & + \mathscr{K}_2 \cdot \sum_{E,E' \in \text{MSIT}} F(E, E') \cdot \frac{l_E \cdot l_{E'} \cdot w_{E'}}{w_E} \\ & + \mathscr{K}_3 \cdot \sum_{E,E' \in \text{MSIT}} F(E, E') \cdot \frac{l_E \cdot l_{E'}}{w_E} \\ & + \mathscr{K}_4 \cdot \sum_{E \in \text{MSIT}} G(E) \cdot \frac{l_E}{w_E} + \mathscr{K}_5 \cdot \sum_{E \in \text{MSIT}} H(E) \cdot \frac{l_E^2}{w_E}, \end{aligned} \tag{7}$$

where

$$\mathscr{K}_0 = \sum_{N_i \in \text{src}(\text{MSIT})} \sum_{N_j \in \text{sink}^i(\text{MSIT})} \lambda^{ij} \cdot K_0^{ij}, \tag{8}$$

$$\mathscr{K}_1 = \sum_{N_i \in \text{src}(\text{MSIT})} \sum_{N_j \in \text{sink}^i(\text{MSIT})} \lambda^{ij} \cdot K_1^{i}, \tag{9}$$

$$F(E, E') = \sum_{N_i \in \text{src}(\text{MSIT})} \sum_{N_j \in \text{sink}^i(\text{MSIT})} \lambda^{ij} \cdot f^{ij}(E, E'), \tag{10}$$

$$G(E) = \sum_{N_i \in \text{src}(\text{MSIT})} \sum_{N_j \in \text{sink}^i(\text{MSIT})} \lambda^{ij} \cdot g^{ij}(E), \tag{11}$$

$$H(E) = \sum_{N_i \in \text{src}(\text{MSIT})} \sum_{N_j \in \text{sink}^i(\text{MSIT})} \lambda^{ij} \cdot h^{ij}(E). \tag{12}$$

Our MSWS problem is aimed at finding the optimal w_Es to minimize the weighted delay formulation Equation (7). Although this weighted delay formulation for multiple sources and multiple sinks is very similar to that for the single source and multiple sinks in Cong and Koh [1994], the coefficient functions F, G, and H have very different properties, which lead to much higher complexity and very different properties for the MSWS problem when compared to the SSWS problem. These properties are discussed in Section 3.

3. PROPERTIES OF OPTIMAL MSWS SOLUTIONS

The SSWS problem under an a priori fixed segment-division was studied in Cong and Leung [1993, 1995], and the polynomial-time optimal algorithm was developed based on a separability, the monotone property, and the dominance property. The presence of multiple sources, however, greatly complicates the wiresizing problem. For example, with multiple sources, even a monotone wiresizing solution is not well defined. Nevertheless, our research has revealed a number of interesting properties of the optimal MSWS solutions under the decomposition of MSITs. Some of them generalize the results on the SSWS problem, and others are unique for the MSWS problem. These properties (presented in this section and Section 4) enable us to apply the algorithms developed in Cong and Leung [1993, 1995] to the MSWS problem to a certain extent and to develop even more efficient algorithms in Section 5.

3.1 Decomposition of an MSIT

When there is only one source in the routing tree, each segment has a unique signal direction and the ancestor-descendant can be defined with respect to the direction. The MSWS problem is most complicated by the fact that, in general, there is no fixed signal direction for a segment. In order to reduce the complexity with the MSWS problem, we decompose an MSIT into the source subtree (SST) and a set of loading subtrees (LSTs) (see Figure 1). The SST[3] is the subtree spanned by all source nodes in the MSIT. After we remove the SST from the MSIT, the remaining segments form a set of subtrees, each of them called an LST. When every pin of an MSIT can be a source at different times, the entire MSIT becomes the SST and there is no LST.

Parallel to the ancestor-descendant relation in an SSIT, the left–right relation is introduced in an MSIT. We choose an arbitrary source as the leftmost node $Lsrc$. The direction of the signal flowing out from $Lsrc$ is defined as the right direction along each segment S. Under such definition, the signal in any LST always flows rightward, but the signal may flow either leftward or rightward in a segment in the SST. The properties of

[3]Note that SST defined in this paper is different from that defined in Cong and Leung [1993, 1995], where SST is used to denote a single stem tree.

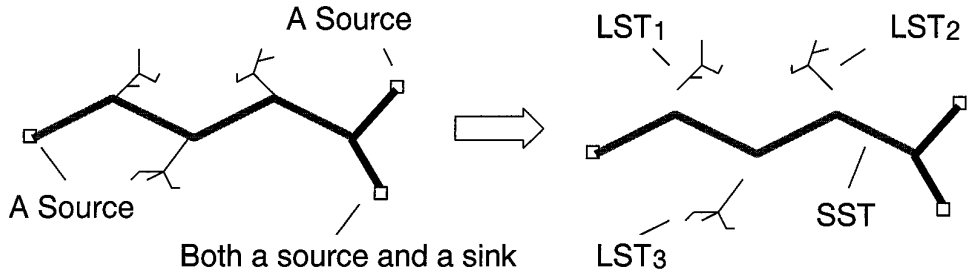

Fig. 1. MSIT decomposed into the source subtree SST and a set of loading subtrees (three LSTs here) branching off from the SST. Dark segments belong to the SST.

optimal MSWS solutions are studied in the context of the MSIT decomposition.

3.2 Properties of Optimal MSWS Solutions

LST Separability.

THEOREM 1. *Given the wire width assignment of the SST, the optimal width assignment for each LST branching off from the SST can be carried out independently. Furthermore, given the wire width assignment of both the SST and a path P originated from the root of an LST, the optimal wire width assignment for each subtree branching off from P can be carried out independently.*

The first part of Theorem 1 is the separability between LSTs. Thus, for the MSIT in Figure 1, the optimal wire widths for LST_1, LST_2, and LST_3 can be computed independently if the wire widths for the SST are given. The second part of Theorem 1 is the separability within an LST, which is the counterpart of the separability in the SSWS problem since an LST can be viewed as an SSIT with its driver located at the branching node from the SST. Because the separability may not hold within the SST, the MSWS problem has much higher complexity than the SSWS problem in general.

LST Monotone Property.

THEOREM 2. *For an MSIT, there exists an optimal wiresizing solution $\mathcal{W}^*$ where the wire widths decrease monotonically rightward within each LST in the MSIT.*

Again, with respect to the analogy between an LST and an SSIT, and replacing the left–right relation in the LST with the ancestor-descendent relation in an SSIT, the LST monotone property is just like the monotone property for the SSWS problem. Because the optimal wiresizing algorithm OWSA developed in Cong and Leung [1993, 1995] for the SSWS problem is based on the separability and the monotone property, according to Theorems 1 and 2, it can be applied independently to each LST when the wire width assignments for the SST are given. Since OWSA is a polynomial-time

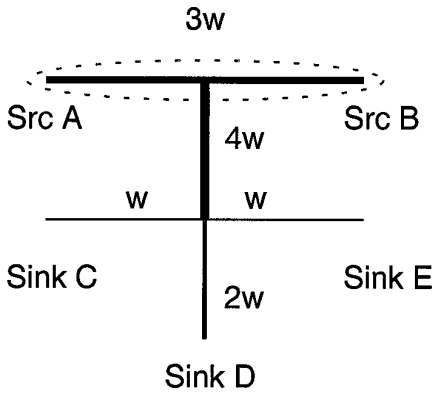

Fig. 2. Optimal wire width assignments for a two-source net with w being the minimum wire width. Dashed curve surrounds the SST. Segments outside curve belong to the LST.

algorithm, the optimal wire widths for the entire MSIT will be computed in the polynomial-time with respect to the given wire widths for the SST.

Furthermore, it is worthwhile emphasizing that the monotone property for the MSWS problem just holds within an LST. The root unisegment in an LST may be wider than the unisegment from which the LST branches. An optimal MSWS solution based on the parameter for the second metal layer (M2) given in Table I is shown in Figure 2. The total wire length is 600 μm. In the optimal solution, the wire width assignment is monotone within the LST, however, the root unisegment of the LST is wider than unisegments in the SST. This example also shows that the monotone property like that in the SSWS problem does not hold for any particular source in an MSIT.

SST Local Monotone Property. Although the signal direction is changeable in the segments of the SST when different sources are active, surprisingly, our study shows the optimal MSWS solutions still satisfy a local monotone property (Theorem 3) given after Lemma 1.

LEMMA 1. *Given an MSIT and a segment S in the MSIT, for any unisegments E_1 and E_2 ($E_1 \neq E_2$) within segment S, $F(E_1, E_2)$ defined in Equation* (10) *is an invariant (denoted $F_l(S)$) if E_1 is left of E_2, and $F(E_1, E_2)$ is another invariant (denoted $F_r(S)$) if E_1 is right of E_2.*

THEOREM 3. *There exists an optimal wiresizing solution for an MSIT, such that the wire widths within each segment are monotone:* (1) *if $F_l(S) > F_r(S)$, the wire widths within S decrease monotonically rightward;* (2) *if $F_l(S) = F_r(S)$, the wire width within S does not change;* (3) *if $F_l(S) < F_r(S)$, the wire widths within S increase monotonically rightward.*

Of course, the local monotone property holds for segments in LSTs, where the $F_l(S)$ is always greater than $F_r(S)$ [in fact, $F_r(S) = 0$] and the wire widths always decrease rightward, just as given by the LST monotone property in an even stronger sense.

Dominance Property.

Definition 1. Given two wiresizing solutions $\mathcal{W}$ and $\mathcal{W}'$, we define $\mathcal{E}$ dominates $\mathcal{W}'$ if $w_E \geq w'_E$ for every unisegment E.

Definition 2. Given a wiresizing solution $\mathcal{W}$ for the routing tree, and any particular unisegment E in the tree, a local refinement on E is defined to be the operation to minimize the objective function Equation (7) by changing only the wire width of E while keeping the wire width assignment of $\mathcal{W}$ on other unisegments unchanged.

THEOREM 4. *Suppose that $\mathcal{W}^*$ is an optimal wiresizing solution for an MSIT. If a wiresizing solution $\mathcal{W}$ dominates $\mathcal{W}^*$, then the wiresizing solution obtained by any local refinement of $\mathcal{W}$ still dominates $\mathcal{W}^*$. Similarly, if $\mathcal{W}$ is dominated by $\mathcal{W}^*$, then the wiresizing solution obtained by any local refinement of $\mathcal{W}$ is still dominated by $\mathcal{W}^*$.*

Although the dominance property was proven based on the ancestor-descendent relation in Cong and Leung [1993, 1995] for the SSWS problem, we proved that it not only holds for the MSWS problem, but is also independent of the ancestor-descendent relation in the SSWS problem, or the left–right relation in the MSWS problem. Theorem 4 enables efficient computations of lower and upper bounds of the optimal wiresizing solution for the MSWS problem by the greedy wiresizing algorithm GWSA [Cong and Leung 1993, 1995] originally developed for the SSWS problem. It applies the local refinement operation iteratively to every unisegment to compute the lower or the upper bound of the optimal wiresizing solution. A much more powerful refinement operation, called the bundled refinement operation, which may compute the lower or the upper bound for a number of unisegments in a single operation, is introduced in Section 4.2.

3.3 Extensions to Multilayer Layout

Up to now, all properties have been discussed under the assumption that all wires lay in the same routing layer. In real layout designs, interconnects are often routed using more than one layer. Similar to the extension made for the SSWS problem in Cong and Leung [1995], the MSWS formulation can be extended to the multilayer cases. In the multilayer formulation, the LST separability and the dominance property still hold. The LST monotone property holds within each layer; that is, there always exists an optimal wiresizing solution such that the wire widths decrease monotonically rightward within each layer for each LST. Furthermore, even in the same layer, if the allowable minimum and maximum wire widths are different from segment to segment due to obstacles in the routing area or reliability considerations, the LST monotone property holds only within segments in the same layer such that these segments have uniform allowable minimum and maximum wire widths. Moreover, it is reasonable to assume that each segment always stays in the same layer and its allowable minimum and maximum wire widths remain unchanged within the segment. In this case,

the local monotone property always holds. Note that all discussions and the bundled refinement property to be presented in Section 4, just as the dominance property, hold for any layer assignment and any allowable minimum or maximum wire width.

4. PROPERTIES OF OPTIMAL MSWS/E SOLUTIONS

Up to now, both the MSWS problem defined in this article and the SSWS problem studied in the literature[4] are only studied in the context of an a priori fixed segment-division. Intuitively, a finer segment-division may lead to a better wiresizing solution. However, it is difficult to choose a proper segment-division. For the best accuracy, a very fine, often uniform segment-division needs to be chosen, which results in high memory usage and computation time due to the large number of unisegments. We now investigate methods to obtain the optimal wiresizing results using a nonuniform and coarser segment-division. A novel contribution of our work is to introduce an MSWS formulation based on a variable segment-division. The segment-division might be finer in some regions but coarser in others. Moreover, we begin with a coarser segment-division and then proceed to a finer one. Theorem 5, to be presented in Section 4.2, justifies this strategy and leads to much more efficient algorithms with the same accuracy when compared with previous work. All properties in this section hold for both the MSWS problem and the SSWS problem, but we concentrate on the MSWS problem since the SSWS problem can be treated as a special case.

4.1. Segment-Division and Bundled-Segment

We assume that *minLength* is a constant determined by the user or the technology such that the wire widths are allowed to change every *minLength* long; in other words, *minLength* is the minimum length that a unisegment can be. Given an MSIT, let $\mathscr{E}_0$ be the segment-division where each unisegment is a segment in the MSIT, and $\mathscr{E}_F$ the uniform segment-division where each unisegment is *minLength* long.[5] Given two segment-divisions $\mathscr{E}$ and $\mathscr{E}'$, if each unisegment in $\mathscr{E}$ corresponds to single or multiple unisegments in $\mathscr{E}'$, we say that $\mathscr{E}'$ is a refinement of $\mathscr{E}$. A segment-division $\mathscr{E}$ is valid only if $\mathscr{E}$ is a refinement of $\mathscr{E}_0$ and the length of every unisegment is a multiple of *minLength*. Clearly, among all valid segment-divisions, $\mathscr{E}_0$ is coarsest and $\mathscr{E}_F$ is finest.

With these definitions, the variable segment-division multisource wiresizing (MSWS/E) problem, can be formulated as follows.

Formulation 2. Given an MSIT, the minimum unisegment length *minLength*, and a set of possible wire width choices, the *MSWS/E* problem for delay minimization is to determine both a segment-division $\mathscr{E}$ and a

[4]Cong and Leung [1993, 1995], Cong and Koh [1994], Sapatnekar [1994], Menezes et al. [1994, 1995], Lillis et al. [1995], Xue and Kuh [1995], Xue et al. [1996].

[5]For simplicity, we assume that the length of any segment in an MSIT is a multiple of *minLength*.

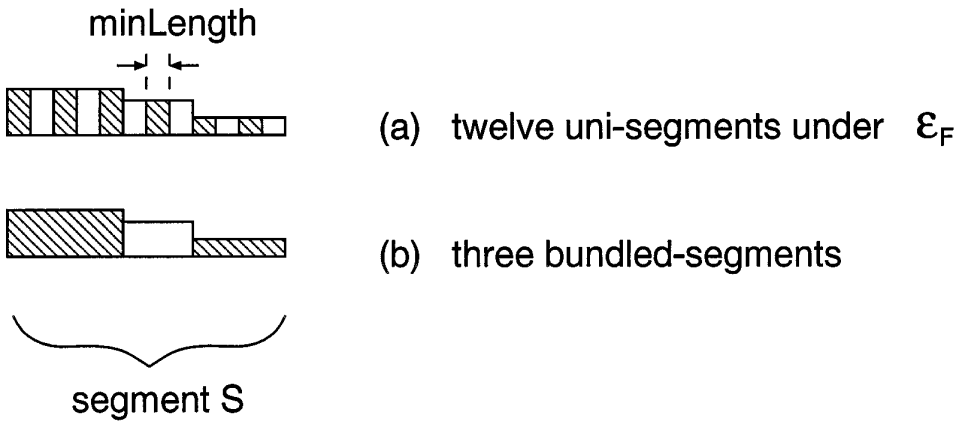

Fig. 3. Optimal wiresizing solution for segment S: (a) under the finest segment-division; (b) under a coarser segment-division with fewer computation costs.

wiresizing solution $\mathcal{W}$, such that the weighted delay $t(MSIT, \mathcal{E}, \mathcal{W})$ is minimized.

Definition 1 is extended to consider the variable segment-division cases.

Definition 3. Given two wiresizing solutions $\mathcal{W}$ and $\mathcal{W}'$, we define $\mathcal{W}'$ dominates $\mathcal{W}$ if $w'_E \geq w_E$ for every unisegment E under the finest segment-division $\mathcal{E}_F$.

The concept of bundled-segment is defined in order to achieve a segment-division as coarse as possible without the loss of wiresizing accuracy.

Definition 4. Given an *MSIT*, a segment S and the finest segment-division $\mathcal{E}_F$, let $E_1, \ldots, E_p$ be a maximal sequence of successive unisegments in S and under $\mathcal{E}_F$ such that all unisegments in this sequence have the same wire width in the optimal wiresizing solution under $\mathcal{E}_F$, we say that these unisegments in the sequence form a bundled-segment.

Figure 3 illustrates the concept of the bundled-segment by showing the optimal wiresizing solution for segment S in an MSIT. It has 12 unisegments under the finest segment-division $\mathcal{E}_F$ (Figure 3.a), but just 3 bundled-segments (Figure 3.b). Clearly, the segment-division defined by the bundled-segments can achieve the same wiresizing solution as that obtained by the finest segment-division $\mathcal{E}_F$. For a long segment or a small *minLength* used in order to achieve a better wiresizing solution, the number of unisegments under the finest segment-division tends to be quite large whereas the number of bundled-segments in the segment is always bounded by a really small constant, as given by the following corollary of the local monotone property (Theorem 3).

COROLLARY 1. *Each segment in an MSIT has at most r bundled-segments where r is the number of possible wire width choices.*

Obviously, using the segment-division defined by the bundled-segments can achieve the required wiresizing solution for the lowest costs. A bundled refinement operation that leads to the computation of the optimal width for a bundled-segment directly, instead of treating it as a sequence of unisegments under the finest segment-division $\mathscr{E}_F$, is presented in the next subsection.

4.2 Bundled Refinement Property

Let $\mathscr{W}$ be a wiresizing solution that dominates the optimal solution $\mathscr{W}^*$, and E be a unisegment under the current segment-division $\mathscr{E}$ and in segment S. Without loss of generality, we assume $F_l(S) \geq F_r(S)$ and treat E as two unisegments E_l and $\overline{E_l}$ during the bundled refinement operation. E_l is the leftmost part of E, with length *minLength* (recall *minLength* is the length for a unisegment in the finest segment-division $\mathscr{E}_F$); $\overline{E_l}$ *is the remaining part of* E. Let $\tilde{w}_{E_l}$ be the locally optimized width for E_l based on the objective function Equation (7) while keeping the width assignment of $\mathscr{W}$ on $\overline{E_l}$ and any unisegment E' other than E. Then, $\tilde{w}_{E_l}$ is regarded as a refined upper bound of the entire unisegment E (not only E_l). This operation is called a bundled refinement operation for the upper bound (BRU).

The rationale for the BRU operation is as follows: if $F_l(S) \geq F_r(S)$, in the optimal solution $\mathscr{W}^*$, E_l is always wider than all unisegments under $\mathscr{E}_F$ in $\overline{E_l}$ (according to the local monotone property). The refinement of an upper bound of $w^*_{E_l}$ is still an upper bound of it (according to the dominance property), thus it also gives an (possibly refined) upper bound of the optimal wire width assignment for any unisegment under $\mathscr{E}_F$ in $\overline{E_l}$. Note that E will not be divided into E_l and $\overline{E_l}$ when performing the BRU operation on unisegments other than E.

Similarly, the bundled refinement operation for the lower bound (BRL) can be defined for a wiresizing solution $\mathscr{W}$ dominated by $\mathscr{W}^*$. Again, assuming $F_l(S) \geq F_r(S)$, we treat E as two unisegments E_r and $\overline{E_r}$. E_r is the rightmost part of E, with length *minLength*; $\overline{E_r}$ is the remaining part of E. Let $\tilde{w}_{E_r}$ be the locally optimized width for E_r based on the objective function Equation (7) while keeping the assignment of $\mathscr{W}$ on $\overline{E_r}$ and any unisegment E' other than E. Then, $\tilde{w}_{E_r}$ is regarded as a refined lower bound of the entire unisegment E.

Concerning the bundled refinement operation, the bundled refinement property similar to the dominance property for the local refinement operation is given as Theorem 5, which leads to the bundled wiresizing algorithm presented in Section 5.1.

THEOREM 5. *Let $\mathscr{W}^*$ be an optimal wiresizing solution under $\mathscr{E}_F$. If a wiresizing solution $\mathscr{W}$ dominates $\mathscr{W}^*$, then the wiresizing solution obtained by any BRU operation on $\mathscr{W}$ under any segment-division $\mathscr{E}$ still dominates $\mathscr{W}^*$. Similarly, if $\mathscr{W}$ is dominated by $\mathscr{W}^*$, then the wiresizing solution obtained by any BRL operation on $\mathscr{W}$ under any segment-division $\mathscr{E}$ is still dominated by $\mathscr{W}^*$.*

5. OPTIMAL MSWS ALGORITHM

5.1 Bundled Wiresizing Algorithm

Based on the dominance property (Theorem 4), the greedy wiresizing algorithm GWSA [Cong and Leung 1993, 1995] originally developed for the SSWS problem is applicable to the MSWS problem. Working on an a priori defined segment-division, GWSA can use local refinement operations to compute the lower or the upper bound of the optimal wiresizing solution starting with the minimum or the maximum wire width assignment, respectively. Based on the bundled refinement property, a new algorithm, the bundled wiresizing algorithm (BWSA) (Figure 4) is proposed to compute the lower and upper bounds of the optimal wiresizing solution for an MSIT. BWSA also starts with the minimum and maximum wire width assignments, but uses bundled refinement operations instead of local refinement operations, and a gradually refined segment-division rather than a fixed one. BWSA achieves the same optimal lower and upper bounds for much less computation costs when compared with GWSA.

Overview. Starting with the coarsest segment-division $\mathscr{E}_0$, we perform BRU and BRL iteratively through an MSIT. We assign the minimum width to all unisegments (in this case, each unisegment is a segment), then traverse MSIT and perform BRL operation on each unisegment. This process is repeated until no improvement is achieved on any unisegment in the last round of traversal. Because the minimum wire width assignment is dominated by the optimal wiresizing solution, according to the bundled refinement property, the resulting wiresizing solution is still dominated by the optimal wiresizing solution and is a lower bound of it. Similarly, we assign the maximum width to all unisegments and perform BRU operations, obtain an upper bound of the optimal wiresizing solution. This is the first pass of BWSA.

After each pass, we check the lower and upper bounds. If there is a gap between the lower and upper bounds for a unisegment (which is called a nonconvergent unisegment) and it is still longer than the minimum unisegment length *minLength*, we divide it into two unisegments of almost equal length (they may differ by *minLength* in order to maintain a valid segment-division), and let each unisegment inherit the lower and upper bounds from the parent. After the refinement of all nonconvergent unisegments, another pass to tighten the lower and upper bounds is carried out by performing bundled refinement operations under the refined segment-division. Note that the bundled refinement is only needed for unisegments that are just refined, because only these unisegments are not convergent.

This BWSA algorithm iterates through a number of passes until we either have the identical lower and upper bounds for all unisegments under current segment-division (in this case we get an optimal wiresizing solution), or each nonconvergent unisegment is *minLength* long.

```
Function gBWSA_L/U(minLength, E, W_lower/W_upper)
  /*
   *  Given the minimum uni-segment length minLength, an segment-division E, a lower/upper
   *  bound W_lower/W_upper of the optimal wiresizing solution, and a set of possible wire widths
   *  {W_1, W_2, ..., W_r}, return a tight lower/upper bound.
   */
  W ← W_lower/W_upper;
  do
    progress ← false;
    for each uni-segment E of E do
      w ← BRL(minLength, E, W_lower, E) or BRU(minLength, E, W_upper, E);
      if w ≠ W(E) then
        progress ← true;    W(E) ← w;
      end if
    end for;
  while progress = true;
  return W;
end Function;

Function SBSR(minLength, E, W_lower, W_upper)
  /*
   *  Selective Binary Segment-division Refinement: Given the minimum uni-segment length
   *  minLength, an segment-division E, a lower bound and an upper bound of the optimal
   *  wiresizing solution, return a refined segment-division.
   */
  E' ← φ;
  for each uni-segment E of E do
    if W_lower(E) ≠ W_upper(E), and E is longer than minLength
      then  divide E into two uni-segments, E' and E'', with (nearly) equal lengths;
            W_lower(E') = W_lower(E'') = W_lower(E);
            W_upper(E') = W_upper(E'') = W_upper(E);
            E' ← E' + {E', E''};
      else  E' ← E' + {E};
    end if;
  end for;
  return E';
end Function;

Function BWSA(E_0, minLength)
  /*
   *  Given the coarsest segment-division E_0, the minimum uni-segment length minLength and a
   *  set of possible wire widths {W_1, W_2, ..., W_r}, return the E_F-tight lower and upper bounds
   *  of the optimal wiresizing solution.
   */
  E' ← E_0;   W_lower ← W_1;   W_upper ← W_r;
  do
    E ← E';
    W_lower ← gBWSA_L(minLength, E, W_lower);
    W_upper ← gBWSA_U(minLength, E, W_upper);
    E' ← SBSR(minLength, E, W_lower, W_upper);
  while E ≠ E'
  return W_lower and W_upper;
end Function;
```

Fig. 4. Bundled wiresizing algorithm (BWSA).

Optimality. In order to discuss the optimality of the lower and upper bounds obtained by the BWSA algorithm, we define the following $\mathcal{E}_F$-tight lower and upper bounds.

Definition 5. If a wiresizing solution $\mathcal{W}$ dominates the optimal solution $\mathcal{W}^*$ and cannot be further refined by any local refinement operation under the finest segment-division $\mathcal{E}_F$, $\mathcal{W}$ is an $\mathcal{E}_F$-tight upper bound. Similarly, $\mathcal{W}$ is an $\mathcal{E}_F$-tight lower bound if $\mathcal{W}$ is dominated by $\mathcal{W}^*$ and cannot be further refined by any local refinement operation under $\mathcal{E}_F$.

It is easy to find that the lower and upper bounds given by the GWSA algorithm are $\mathcal{E}_F$-tight. In addition, it is worthwhile mentioning that there may be more than one $\mathcal{E}_F$-tight upper (or lower) bound for a $\mathcal{W}^*$. An experimental example of nonunique $\mathcal{E}_F$-tight bounds is given in Section 6.2.

With this definition, we proved the following important result concerning the optimality of the BWSA algorithm.

THEOREM 6. *The lower and upper bounds provided by BWSA are $\mathcal{E}_F$-tight.*

Basically, Theorem 6 suggests that the quality of the wiresizing solutions obtained by the BWSA algorithm starting from the coarsest segment-division is as good as those obtained by the GWSA algorithm using the finest segment-division $\mathcal{E}_F$.

Complexity. Recall that our MSWS/E problem aims to find the optimal wiresizing solution for every wire that is *minLength* long. In order to achieve the required accuracy, the finest segment-division $\mathcal{E}_F$ where each unisegment is *minLength* long must be used by GWSA, whereas BWSA can determine a proper, usually coarser, segment-division during the wiresizing procedure. If we use *minLength* as the wire length unit, the total wire length n is a natural metric to measure the problem size. We proved the following Theorem 7.

THEOREM 7. *Given an MSIT and r wire width choices, if the total wire length is n when regarding minLength as the length unit, both GWSA and BWSA have the worst-case complexity of $O(n^3 \cdot r)$ for the MSWS/E problem.*

It is worthwhile emphasizing that the final unisegment produced by BWSA is often much longer than *minLength* and BWSA runs much faster than GWSA in the practice, which is supported by extensive experiments in Section 6.2 and Cong and He [1995b]. In fact, because BWSA runs much faster than GWSA and obtains the same tight lower and upper bounds as those obtained by GWSA, we always use BWSA instead of GWSA. Furthermore, due to the fact that the BWSA computes both lower and upper bounds of the optimal wiresizing solution based on the bundled refinement property, we can tell easily when the optimal wire widths are achieved for those unisegments where their lower and upper bounds meet, so that we do not have to further refine the segment-division for them. Similar segment-division refinement schemes may not be used optimally in other wiresizing

methods[6] until there is an easy way to determine that the current wiresizing solution is the optimal wiresizing solution or part of it belongs to the optimal wiresizing solution.

5.2 Optimal Wiresizing Algorithm Using Bundled Refinement

Given an MSIT, BWSA can be used to compute the $\mathscr{E}_F$-tight lower/upper bounds of the optimal wiresizing solution. If the lower and upper bounds meet, which is very likely in practice, we get the optimal wiresizing solution immediately. Otherwise, the optimal solution shall be found between the lower and upper bounds. Because of the LST separability and the LST monotone property, OWSA, originally developed for the SSWS problem in Cong and Leung [1993, 1995], can be used independently for every LST with respect to the given wire width assignments for the SST. However, since the separability in SST does not hold in general, the optimal wire width assignments for nonconvergent unisegments in the SST will be found by enumeration between the $\mathscr{E}_F$-tight lower and upper bounds and subject to the local monotone property. Thus, the optimal wiresizing algorithm using bundled refinement (OWBR algorithm) has been developed, which works like the following.

(1) Compute the $\mathscr{E}_F$-tight lower and upper bounds by BWSA;
(2) Enumerate the wire width assignments for the SST between the $\mathscr{E}_F$-tight lower and upper bounds and subject to the local monotone property;
(3) Apply OWSA independently to each LST during the enumeration of wire width assignments for the SST and subject to the $\mathscr{E}_F$-tight lower and upper bounds.

Our experiments show that BWSA gives the convergent bounds on all unisegments in an MSIT for almost all cases. For those cases that have nonconvergent unisegments, the percentage of nonconvergent unisegments is very small. Moreover, the gap between the lower and upper bounds on each nonconvergent unisegment is also very small (usually being one in our experiments). Therefore, OWBR runs very fast in practice. Note that the OWBR algorithm can be extended to the multilayer case the same as the extension of the OWSA algorithm in Cong and Leung [1995]. Experimental results with multilayer MSIT designs are presented in Section 6.

6. EXPERIMENTAL RESULTS

We have implemented the OWBR algorithm in ANSI C for the Sun SPARC station environment and tested our algorithm on multisource nets extracted from the multilayer layout of an Intel high-performance microprocessor. In this section, we present the comparison of different wiresizing solutions, the comparison between the BWSA algorithm and the GWSA

[6]Sapatnekar [1994], Menezes et al. [1994, 1995], Xue and Kuh [1995], Xue et al. [1996], Lillis et al. [1995].

Table I. Parameters Based on MCNC 0.5 μm Submicron CMOS Technology

Metal Layer:	M1	M2
Wire Resistance ($\Omega/\square$):	0.068	0.044
Wire Capacitance (area) ($aF/\mu m^2$):	130.6	41.3
Fringing Capacitance (2 sides) ($aF/\mu m$):	161.9	150

algorithm, and the fidelity study of the Elmore delay model versus the SPICE-computed delay to justify our formulation based on the Elmore delay model.

The parameters used in our experiments are summarized in Table I. These parameters are based on the 0.5 μm CMOS technology of the North Carolina Microelectronic Center (MCNC) designers manual. Since only parameters about the first and the second metal layers (M1 and M2) are available, we only use layers M1 and M2 in our experiments. The wire width choices in each layer are $\{W, 2W, 3W, 4W, 5W\}$ with W being the minimum allowable wire width in the layer. Note that our algorithms are still valid if wire widths are not multiples of the minimum width. The minimum unisegment length *minLength* is set to be 10 μm. We assume that the driver is an inverter, its p-type transistor is 105.9μm-wide and n-type 53.5μm-wide. Its effective resistance is 156Ω based on SPICE simulation. We model the driver as a resistor of this value during the wiresizing procedure. In addition, the loading capacitance in every loading is set to be 3.720 fF. Note that both our formulation and implementation can handle cases where different sources have different driver resistances and different sinks have different loading capacitances.

6.1 Comparison Among Different Wiresizing Solutions

We report SPICE-computed delays instead of calculated Elmore delay values in the comparison among different wiresizing solutions. For the SPICE simulation in this paper (the driver is modeled by the SPICE Level-3 MOSFET model given in MCNC designers manual), and every wire of *minLength* long (10μm) by an RC circuit. The use of SPICE simulation results not only shows the quality of our MSWS solutions, but also verifies the validity of our interconnect modeling and the correctness of our MSWS problem formulation.

The test suite used for our algorithms comprises real multisource nets provided by Intel [Chan 1995]. These nets were extracted from the top-level floor-plan of a high-performance microprocessor. Most pins of these nets can serve as both sources and sinks at different times, and almost all pairs between sources and sinks (excluding feed-through pins) are timing critical. We use the 1-Steiner tree algorithm [Kahng and Robins 1992] to route these nets. Table II summarizes the routing trees for these nets.

We applied our OWBR algorithm to these MSITs. First, we assume all wires in M2; then, we assume all wires parallel to the X-axis in M1, the

Table II. Routing Trees for Extracted Multisource Nets

	total pin number	total segment number	total wire length (μm)
net1	3	4	3600
net2	4	6	6600
net3	9	13	10070
net4	4	5	10570
net5	11	10	16980
net6	19	19	31980

rest in M2. Let *min_width* be the wiresizing solution with minimum wire width W everywhere, and *opt_msws* the multisource wiresizing solution given by our OWBR algorithm. Also, let wire length denote the total wire length of a routing tree, and normalized area denote the area ratio of the wiresizing solution versus the *min_width* wiresizing solution, which is equivalent to the average wire width if the minimum wire width W is scaled to 1. Both average delay and maximum delay are only measured on critical source-sink pairs. In these experiments, we assign $\lambda^{ij} = 1$ for a critical source-sink pair and $\lambda^{ij} = 0$ otherwise. Thus, the objective in Equation (7) is equivalent to the average delay among critical source-sink pairs. Comparisons among different wiresizing solutions are shown in Table III. In terms of the average delay, the objective of our MSWS formulation, the *opt_msws* solutions consistently outperform the *min_width* solutions. The delay reduction is up to 23.5% and 12.6% for the single-layer and multilayer cases, respectively. It is interesting to observe that although the average delay is our objective, experimental results show that this formulation reduces the maximal delay substantially (only in one example, *opt_msws* loses 0.69% in terms of the maximum delay, but still wins in terms of the average delay). The maximum delay reduction is up to 36.3% and 37.8% for the single-layer and multi-layer cases, respectively. In addition, the delay reduction for nets with larger spans is observed to be more significant. It often happens in our experiments that the optimal wiresizing solution for nets with fewer pins and shorter total wire lengths is simply the minimum wire width solution. In general, the optimal wiresizing is more effective for global nets with more pins and longer total wire lengths.

6.2 Speedup Using Variable Segment-Division

We applied both BWSA and GWSA algorithms to the test suite of Intel nets. Because the time for BWSA to compute the $\mathscr{E}_F$-tight lower and upper bounds for most nets in the test suite is too small to measure, we compared the total running time. In Table IV, the BWSA-based algorithm is just OWBR, that is, BWSA to compute $\mathscr{E}_F$-tight lower and upper bounds, followed by enumerating for the SST and OWSA for LSTs. The GWSA-based algorithm is just to replace BWSA by GWSA in the OWBR scheme. The BWSA-based algorithm is observed to run more than 100× faster than the GWSA-based algorithm.

Table III. Multisource Wiresizing Result Comparison

All Wires in M2					
	Normalized Area	Average Delay (ns)		Maximum Delay (ns)	
	opt_msws	min_width	opt_msws	min_width	opt_msws
net1	1.000	0.1198	0.1198(0.0%)	0.1224	0.1224 (0.0%)
net2	1.044	0.2004	0.1994(-0.50%)	0.2572	0.2567 (-0.2%)
net3	1.475	0.3504	0.3241(-7.5%)	0.5230	0.4025 (-23.0%)
net4	2.000	0.5007	0.3846(-23.2%)	0.5853	0.4873 (-16.7%)
net5	1.775	0.6375	0.5711(-10.4%)	0.9496	0.7635 (-19.6%)
net6	2.706	1.8968	1.4512(-23.5%)	3.4505	2.1979 (-36.3%)
X-axis Wires in M1 and Y-axis Wires in M2					
	Normalized Area	Average Delay (ns)		Maximum Delay (ns)	
	opt_msws	min_width	opt_msws	min_width	opt_msws
net1	1.000	0.1198	0.1198(0.0%)	0.1224	0.1224 (0.0%)
net2	1.044	0.3250	0.3245(-0.15%)	0.3777	0.3803 (+0.69%)
net3	2.000	0.5336	0.4983(-6.61%)	0.8583	0.7853 (-8.51%)
net4	2.000	0.5514	0.5282(-4.54%)	0.8009	0.7000 (-12.6%)
net5	1.775	0.6445	0.5850(-9.23%)	0.9447	0.7623 (-19.3%)
net6	3.224	2.2752	1.9885(-12.6%)	4.3826	2.7238 (-37.8%)

Table IV. Total Running Time Comparison

	net1	net2	net3	net4	net5	net6
GWSA-based algorithm (s)	0.07	8.18	172.37	15.67	38.10	227.92
BWSA-based algorithm (s)	0.07	0.15	0.37	0.37	0.97	3.37
Speedup factor of BWSA-based algorithm	1	54.5	465.8	42.3	39.3	67.63

It is worthwhile mentioning that BWSA gives identical $\mathscr{E}_F$-tight lower and upper bounds for net3 whereas GWSA does not, which is also an example of existence of multiple $\mathscr{E}_F$-tight bounds for the optimal solution as mentioned in Section 5.1. Also note that, in the case of OWBR, the total running time is not dominated by the time to compute $\mathscr{E}_F$-tight lower and upper bounds: one reason is that the current implementation builds the data structure for the finest segment-division even if the bundled refinement does not need it at all. Thus, the total running time still can be further reduced in future implementation without building the data structure for the finest segment-division.[7]

6.3 Fidelity of the Elmore Delay Model

The concept of fidelity for the Elmore delay model was introduced by Boese et al. [1993] for the routing tree topology optimization to measure if an optimal or near-optimal solution selected according to the Elmore delay model is nearly optimal according to the actual delay (e.g., computed using SPICE). We investigate the fidelity of the Elmore delay model for the

[7]It has been done in Cong and He [1995c, 1996].

Table V. Average Differences in Ranking and SPICE-Computed Delay Based on 0.5 μm CMOS Technology

	Overall (1,000)		Best-100 (Elmore Delay)		Best-10 (Elmore Delay)	
	Ranking Difference	Delay Difference	Ranking Difference	Delay Difference	Ranking Difference	Delay Difference
net1	23.61	0.1048%	11.36	0.0150%	2.800	0.0017%
net2	121.7	0.7223%	69.81	0.1007%	23.10	0.0173%
net3	53.50	0.3264%	33.31	0.1300%	15.40	0.0508%
net4	170.7	0.8517%	54.54	0.0410%	1.400	0.0490%
net5	38.52	0.1462%	14.52	0.0580%	0.900	0.0012%
net6	54.27	0.1812%	25.45	0.0286%	2.000	0.0026%

optimal wiresizing problem, that is, to find out how good the solution given by our optimal wiresizing formulation based on the Elmore delay model is in terms of the real delay.

We measure the fidelity again on the test suite of Intel nets and assume all wires in the M2 layer. Since the number of total wiresizing solutions is prohibitively large to enumerate, we randomly generate 1,000 wiresizing solutions for every MSIT. In a solution, a random wire width is assigned for every wire *minLength* long (10 μm). We obtain both the weighted average Elmore delay and the weighted average 50% delay computed by SPICE for each solution and then rank the 1,000 solutions for each MSIT, using the technique similar to Boese et al. [1993]: first rank solutions according to their weighted Elmore delays; then rank them according to their weighted SPICE-computed delay. The absolute difference between the two rankings of a wiresizing solution is its ranking difference and we average ranking differences over 1,000 solutions for every MSIT. In order to know how large the SPICE-computed delay difference may be with respect to the average ranking difference, delay difference is computed in the following way: let the average ranking difference be d. For a wiresizing solution whose SPICE-computed delay ranking is i, we compute the relative difference between the $(i + d)$th and ith SPICE-computed delays, as well as that between the $(i - d)$th and ith SPICE-computed delays. Between the two values, the one with the larger absolute value is defined as the delay difference for the average ranking difference d. We average delay differences over 1,000 solutions for every MSIT.

The average ranking differences and the associated average delay differences are given in Table V. Let us take net1 as an example to show how good the optimal solution selected according to the Elmore delay model might be. The average rank difference for 1,000 wiresizing solutions is 23.61. Thus, the optimal solution selected according to the Elmore delay, on average, might be $\lceil 23.61 \rceil$ away from the top one in the ranking according to SPICE-computed delays. Since the ranking difference of $\lceil 23.61 \rceil$ accounts for only 0.1448% SPICE-computed delay difference, on average, the optimal solution selected according to Elmore delay is only 0.1448% worse than the

optimal one selected according to the SPICE-computed delays, when delays of both solutions are measured by SPICE simulation.[8]

Over the 1,000 random solutions for each net, the average ranking differences are between 23.61 and 170.7, and average delay differences between 0.1648% and 0.8517%. In addition, we measure the average delay difference for the best-100 and best-10 wiresizing solutions according to the Elmore delay model for each random solution set, respectively. It is interesting to find that the better the wiresizing solutions according to the Elmore delay model, the less the average delay difference they have. Taking net1 as an example, the average delay difference is 0.1048% for the 1,000 solutions, but only 0.0150% for the best-100, and even less, 0.0017% for the best-10. It implies that, in general, in the area near the optimum in the solution space, the Elmore delay model has an even higher fidelity.

Based on data of the best-10 in every random solution set, the optimal wiresizing solution selected according to the Elmore delay model is less than 0.06% worse than the optimal solution selected according to the SPICE-computed delay.[9] Thus, we believe that the Elmore delay model has really high fidelity for wiresizing optimization; that is, the optimal solution selected according to the Elmore delay model is also the optimal solution or nearly the optimal solution selected according to the SPICE-computed delay. Note that the inductance is not taken into consideration in our SPICE simulation, since the inductive effect is negligible under the current CMOS technology. The higher-order delay model used in Menezes et al. [1994, 1995] does not consider the inductance, either.

7. CONCLUSIONS AND FUTURE WORK

The results in this paper have shown convincingly that proper sizing of the wire segments in multisource nets can lead to significant reduction in the interconnect delay. We have also developed an efficient wiresizing algorithm named the BWSA algorithm. It achieves the same wiresizing solution as the GWSA algorithm [Cong and Leung 1993, 1995], but runs 100× time faster and uses much less memory space. Thus, the BWSA algorithm shall be used instead of the GWSA algorithm, not only for the multisource wiresizing problem, but also for the single-source wiresizing problem [Cong and Leung 1993, 1995] and the simultaneous driver and wiresizing problem [Cong and Koh 1994]. Compared to the minimum wire width solution, the optimal wiresizing solution obtained by our algorithm reduces the average delay by up to 23.5% and the maximum delay by up to 37.8%, respectively. It takes several seconds to obtain the optimal wiresizing solution for the

[8]Note that the Elmore delay value of the optimal solution selected according to the Elmore delay model is often quite different from the SPICE-computed delay of the same solution, with 24% error for the optimal wiresizing solution for net1.

[9]We also enumerate the wiresizing solutions for net1 and net2 by assuming that each segment in the routing tree has a uniform wire width. Even higher fidelity is observed when compared with this set of random wiresizing experiments. For these two nets, the Elmore delay model gives the best-5 solutions the same as those given by the SPICE-computed delay.

largest example in our test suite extracted from a high-performance Intel microprocessor.

Extensions have been made in Cong and He [1995c, 1996]. By revealing the dominance property for a class of optimization problems named CH-posynomial programs, the BWSA algorithm has been generalized to approach the simultaneous transistor and interconnect sizing (STIS) problem. The simultaneous driver and wiresizing problem (or the simultaneous buffer and wiresizing problem) can be solved as a simple case of the STIS problem. Different from the simultaneous driver and wiresizing formulation [Cong and Koh 1994] applicable only to the single-source nets, it has been used to further reduce the delay for multisource nets by sizing drivers and wires simultaneously. Furthermore, the STIS formulation uses a transistor delay model with consideration of the waveform slope effect. It is more accurate than the fixed-value resistor model used in this paper and in Cong and Leung [1993, 1995], Cong and Koh [1994], Sapatnekar [1994], and Chen et al. [1996a, b]. Moreover, the STIS formulation can combine delay and area (or power) minimizations and a smooth delay and area tradeoff has been yielded. However, our MSWS formulation does not consider the coupling between wires, which becomes important when the technology moves into the deep submicron. We plan to develop an efficient wiresizing algorithm to take account of the coupling effect.

Another wiresizing optimization objective is to minimize the maximum delay in interconnects under the Elmore delay model. If we assume the single-source net and the fixed-value resistor model for the driver, approaches in Sancheti and Sapatnekar [1994], Menezes et al. [1995], and Chen et al. [1996a] are able to achieve the optimal continuous wiresizing solution, and the approach in Lillis et al. [1995] is able to achieve the optimal discrete wiresizing solution. It is worthwhile mentioning that the approach in Chen et al. [1996a] is based on a Lagrangian relaxation procedure to iteratively apply the weighted delay minimizations (same as that in Cong and Leung [1993, 1995] and similar to our formulation). It adjusts the weight assignments after each iteration until the optimal weight assignments are achieved to minimize the maximum delay by using the weighted delay minimization. In order to minimize the maximum delay for multisource nets, the optimal continuous solution might be achieved by extensions of approaches in Sancheti and Sapatnekar [1994], Menezes et al. [1994a], and Chen et al. [1996a]. However, the optimal algorithm for obtaining the discrete solution is still open. We have shown experimentally that our weighted delay formulation could reduce the maximum delay very well. More work is planned to find out whether an optimal algorithm exists.

The topologies of MSITs may affect the delay reductions that can be achieved by the optimal wiresizing even though our OWBR algorithm is able to achieve the optimal wiresizing solution for any MSIT topology. For single-source nets, simultaneous tree construction and wiresizing has been explored very recently in Lillis et al. [1996] and Okamoto and Cong [1996]. However, the question of how to combine the routing tree construction and

wiresizing to achieve the largest delay reduction for multisource nets is still open. We plan to study the problem in the future.

APPENDIX

Since the proofs for the LST separability (Theorem 1), the LST monotone property (Theorem 2), the dominance property (Theorem 4), and Theorem 7 concerning the complexities of GWSA and BWSA algorithms are similar to those in Cong and Leung [1995], the full proofs of these theorems are given in Cong and He [1995b]. In this appendix, we first discuss the properties for the coefficient functions and then prove the SST local monotone property (Theorem 3), the bundled refinement property (Theorem 5), and Theorem 6 concerning the optimality of the BWSA algorithm.

Properties of Coefficient Functions

Careful study of the definitions of f^{ij}, g^{ij}, and h^{ij} in Equations (4)–(6), as well as F, G, and H in Equations (10)–(12) reveals Lemma 1 (presented in Section 3) and the following Lemmas 2 and 3 for the coefficient functions F, G, and H.

LEMMA 2. *Given an MSIT and a segment S in the MSIT, for any unisegments E and E', if E is in segment S, and E' in segment S' ($\neq S$), $F(E, E')$ is an invariant (denoted $F(S, S')$).*

LEMMA 3. *Given an MSIT and a segment S in the MSIT, for any unisegment E within segment S, $G(E)$ and $H(E)$ are invariants (denoted $G(S)$ and $H(S)$, respectively).*

Although the coefficient functions F, G, and H are defined for unisegments in Equations (10)–(12), Lemmas 1–3 enable us to compute these functions based on segments rather than unisegments. Because the number of segments in an MSIT may be much smaller than the number of unisegments in the MSIT, we can compute these coefficient functions for much reduced costs. These coefficient functions are computed before the wiresizing procedure and viewed as constants during the wiresizing procedure.

Proof of Theorem 3

We prove Theorem 3 (the SST local monotone property) based on a series of lemmas. For simplicity, we assume the finest segment-division in this proof. Recall that a unisegment under any valid segment-division corresponds to single or multiple unisegments in the finest segment-division; it is easy to verify that the local monotone property holds for any valid segment-division if and only if it holds for the finest segment-division.

LEMMA 4. *Given an MSIT and a segment S in the MSIT, if E_l and E_r are two adjacent unisegments within segment S and E_l is just left of E_r, then*

$$F(E_l, E) = F(E_r, E) \quad \text{if } E \neq E_l \neq E_r,$$
$$F(E, E_l) = F(E, E_r) \quad \text{if } E \neq E_l \neq E_r.$$

PROOF. There are two cases for Lemma 4.

Case 1. E is in segment S. According to Lemma 1,
if E is right of E_l (as well as E_r), $F(E_l, E) = F(E_r, E) = F_l(S)$ and $F(E, E_l) = F(E, E_r) = F_r(S)$;
if E is left of E_l (as well as E_r), $F(E_l, E) = F(E_r, E) = F_r(S)$ and $F(E, E_l) = F(E, E_r) = F_l(S)$.

Case 2. E is in segment S' ($\neq S$). According to Lemma 2,
$F(E_l, E) = F(E_r, E) = F(S, S')$ and $F(E, E_l) = F(E, E_r) = F(S', S)$. □

LEMMA 5. *Given an MSIT and a segment S in the MSIT, let E_l and E_r be unisegments in a segment S with E_l just left of E_r. Concerning the optimal wiresizing solution, if $F_l(S) > F_r(S)$, then E_l cannot be narrower than E_r; if $F_l(S) < F_r(S)$, then E_l cannot be wider than E_r.*

PROOF. Let M = MSIT − {E_l, E_r} and $\bar{\mathcal{W}}_M$ be the wiresizing solution defined on M by $\mathcal{W}$. The objective function (7) can be written as:

$$
\begin{aligned}
& t(\mathrm{MSIT}, \mathcal{E}, \mathcal{W}) \\
&= t(M, \mathcal{E}, \bar{\mathcal{W}}_M) + \mathcal{K}_1 \cdot (w_{E_l} + w_{E_r}) \\
&\quad + \mathcal{K}_2 \cdot \sum_{E' \in \mathrm{MSIT}} F(E_l, E') \cdot \frac{w_{E'}}{w_{E_l}} + \mathcal{K}_2 \cdot \sum_{E \in \mathrm{MSIT}} F(E, E_l) \cdot \frac{w_{E_l}}{w_E} \\
&\quad + \mathcal{K}_2 \cdot \sum_{E' \in \mathrm{MSIT}} F(E_r, E') \cdot \frac{w_{E'}}{w_{E_r}} + \mathcal{K}_2 \cdot \sum_{E \in \mathrm{MSIT}} F(E, E_r) \cdot \frac{w_{E_r}}{w_E} \\
&\quad + \mathcal{K}_3 \cdot \sum_{E' \in \mathrm{MSIT}} F(E_l, E') \cdot \frac{1}{w_{E_l}} + \mathcal{K}_4 \cdot G(E_l) \cdot \frac{1}{w_{E_l}} \\
&\quad + \mathcal{K}_5 \cdot H(E_l) \cdot \frac{1}{w_{E_l}} + \mathcal{K}_3 \cdot \sum_{E' \in \mathrm{MSIT}} F(E_r, E') \cdot \frac{1}{w_{E_r}} \\
&\quad + \mathcal{K}_4 \cdot G(E_r) \cdot \frac{1}{w_{E_r}} + \mathcal{K}_5 \cdot H(E_r) \cdot \frac{1}{w_{E_r}}.
\end{aligned}
\tag{13}
$$

Let $\mathcal{W}^*$ be the optimal wiresizing solution. After swapping the width assignments for E_l and E_r with respect to $\mathcal{W}^*$ (see Figure 5), we denote the resulting wiresizing solution $\mathcal{W}^*/E_l, E_r$: $w_{E_l} \leftrightarrow w_{E_r}$.

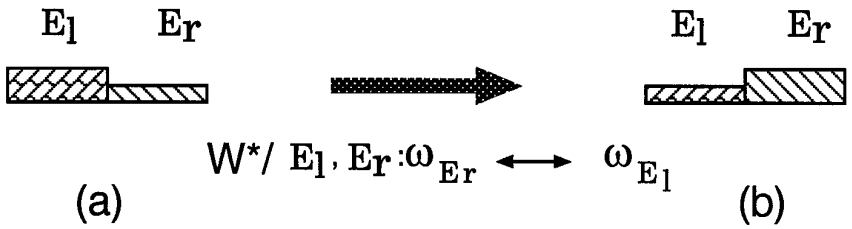

Fig. 5. (a) Width assignments for E_l and E_r in optimal solution $\mathcal{W}^*$; (b) wiresizing solution obtained by swapping width assignments for E_1 and E_r.

According to Lemmas 3 and 4,

$$G(E_l) = G(E_r) = G(S) \tag{14}$$

$$H(E_l) = H(E_r) = H(S) \tag{15}$$

$$F(E_l, E) = F(E_r, E) \qquad \text{if } E \neq E_r \neq E_l \tag{16}$$

$$F(E, E_l) = F(E, E_r) \qquad \text{if } E \neq E_r \neq E_l; \tag{17}$$

thus,

$$\begin{aligned} & t(\text{MSIT}, \mathcal{E}, \mathcal{W}^*/E_l, E_r: w_{E_l} \leftrightarrow w_{E_r}) - t(\text{MSIT}, \mathcal{E}, \mathcal{W}^*) \\ &= \mathcal{H}_2 \cdot F(E_l, E_r) \cdot \left(\frac{w^*_{E_l}}{w^*_{E_l}} - \frac{w^*_{E_r}}{w^*_{E_l}}\right) + \mathcal{H}_2 \cdot F(E_r, E_l) \cdot \left(\frac{w^*_{E_r}}{w^*_{E_l}} - \frac{w^*_{E_l}}{w^*_{E_r}}\right) \\ &\quad + \mathcal{H}_3 \cdot F(E_l, E_r) \cdot \left(\frac{1}{w^*_{E_l}} - \frac{1}{w^*_{E_l}}\right) + \mathcal{H}_3 \cdot F(E_r, E_l) \cdot \left(\frac{1}{w^*_{E_l}} - \frac{1}{w^*_{E_r}}\right) \\ &= \{F(E_l, E_r) - F(E_r, E_l)\} \cdot \{w^*_{E_l} - w^*_{E_l}\} \cdot \left\{\mathcal{H}_2 \cdot \frac{(w^*_{E_l} + w^*_{E_l}) + \mathcal{H}_3}{w^*_{E_l} \cdot w^*_{E_l}}\right\}. \end{aligned} \tag{18}$$

We know that $(\mathcal{H}_2 \cdot (w^*_{E_l} + w^*_{E_r}) + \mathcal{H}_3)/w^*_{E_l} \cdot w^*_{E_r}) > 0$ and Equation (18) ≥ 0 since $\mathcal{W}^*$ is the optimal solution. Clearly, if $F_l(S) > F_r(S)$, according to Lemma 1, $F(E_l, E_r) > F(E_r, E_l)$, then we have $w^*_{E_l} \geq w^*_{E_r}$. Similarly, if $F_l(S) < F_r(S)$, then $F(E_l, E_r) < F(E_r, E_l)$, so we have $w^*_{E_l} \leq w^*_{E_r}$. As a result, Lemma 5 holds. □

By applying Lemma 5 to any adjacent unisegments in a segment, we obtain Lemma 6.

LEMMA 6. *Given an MSIT and a segment S in the MSIT, concerning the optimal wiresizing solution, if $F_l(S) > F_r(S)$, then the wire widths within*

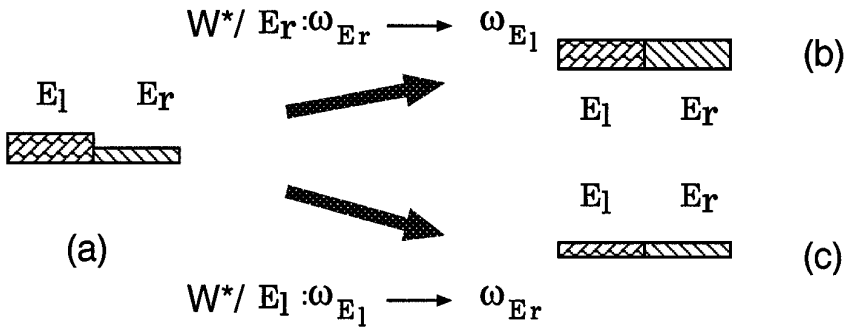

Fig. 6. Width assignments for E_l and E_r in optimal solution $\mathcal{W}^*$; (b) wiresizing solution obtained by replacing width of E_r with that of E_l; (c) wiresizing solution obtained by replacing width of E_l with that of E_r.

segment S decrease monotonically rightward. Similarly, they increase monotonically rightward if $F_l(S) < F_r(S)$.

LEMMA 7. *Given an MSIT and any segment S in the MSIT, if* $F_l(S) = F_r(S)$*, there exists an optimal wiresizing such that all unisegments in segment S have the same wire width.*

PROOF. Assume that Lemma 7 fails for an MSIT, then for any optimal solution $\mathcal{W}^*$, there must exist two unisegments in a segment S of the MSIT such that E_l is just left of E_r and $w^*_{E_l} \neq w^*_{E_r}$. Since $\mathcal{W}^*$ is optimal, the increase in the objective function when we change the width of E_r in $\mathcal{W}^*$ from $w^*_{E_r}$ to $w^*_{E_l}$ (see Figure 6.b), by using Equation (13), is

$$\begin{aligned}
\Delta t1 &= t(\text{MSIT}, \mathcal{E}, \mathcal{W}^*/E_r{:}w_{E_r} \rightarrow w_{E_l}) - t(\text{MSIT}, \mathcal{E}, \mathcal{W}^*) \\
&= \mathcal{H}_1 \cdot (w_{E_l} - w_{E_r}) + \mathcal{H}_2 \cdot \sum_{E' \in \text{MSIT}} F(E_r, E') \cdot \left(\frac{w_{E'}}{w_{E_l}} - \frac{w_{E'}}{w_{E_r}} \right) \\
&\quad + \mathcal{H}_2 \cdot \sum_{E \in \text{MSIT}} F(E, E_r) \cdot \left(\frac{w_{E_l}}{w_E} - \frac{w_{E_r}}{w_E} \right) \qquad (19) \\
&\quad + \mathcal{H}_3 \cdot \sum_{E' \in \text{MSIT}} F(E_r, E') \cdot \left(\frac{1}{w_{E_l}} - \frac{1}{w_{E_r}} \right) \\
&\quad + \mathcal{H}_4 \cdot G(E_r) \cdot \left(\frac{1}{w_{E_l}} - \frac{1}{w_{E_r}} \right) + \mathcal{H}_5 \cdot H(E_r) \cdot \left(\frac{1}{w_{E_l}} - \frac{1}{w_{E_r}} \right) \geq 0.
\end{aligned}$$

Similarly, the increase in the objective function when we change the width of E_l in $\mathcal{W}^*$ from $w^*_{E_l}$ to $w^*_{E_r}$ (see Figure 6c) is:

$$\Delta t2 = t(\text{MSIT}, \mathcal{E}, \mathcal{W}^*/E_l\text{: } w_{E_l} \rightarrow w_{E_r}) - t(\text{MSIT}, \mathcal{E}, \mathcal{W}^*)$$

$$= \mathcal{H}_1 \cdot (w_{E_r} - w_{E_l}) + \mathcal{H}_2 \cdot \sum_{E' \in \text{MSIT}} F(E_l, E') \cdot \left(\frac{w_{E'}}{w_{E_r}} - \frac{w_{E'}}{w_{E_l}} \right)$$

$$+ \mathcal{H}_2 \cdot \sum_{E \in \text{MSIT}} F(E, E_l) \cdot \left(\frac{w_{E_r}}{w_E} - \frac{w_{E_l}}{w_E} \right)$$

$$+ \mathcal{H}_3 \cdot \sum_{E' \in MSIT} F(E_l, E') \cdot \left(\frac{1}{w_{E_r}} - \frac{1}{w_{E_l}} \right)$$

$$+ \mathcal{H}_4 \cdot G(E_l) \cdot \left(\frac{1}{w_{E_r}} - \frac{1}{w_{E_l}} \right) + \mathcal{H}_5 \cdot H(E_l) \cdot \left(\frac{1}{w_{E_r}} - \frac{1}{w_{E_l}} \right) \geq 0. \tag{20}$$

Recall Equations (14)–(17); it is not difficult to verify the following,

$$\Delta t1 + \Delta t2 = 0. \tag{21}$$

According to Equations (19)–(21), $\Delta t1 = \Delta t2 = 0$. That is, both $\mathcal{W}^*/E_r$: $w_{E_r} \rightarrow w_{E_l}$ and $\mathcal{W}^*/E_l$: $w_{E_l} \rightarrow w_{E_r}$ are optimal.

Therefore, if there is an optimal solution $\mathcal{W}^*$ where the wire widths are not uniform in a segment S, let w^*_l be the wire width of the leftmost unisegment in S, from left to right, we can successively replace the wire width for every unisegment in S by w^*_l, without increase in the objective function. That is, the resulting wiresizing solution is an optimal wiresizing solution such that the wire widths are uniform in segment S. □

In conclusion, we obtain the SST local monotone property (Theorem 3) by combining Lemmas 6 and 7.

Proof of Theorem 5

In order to prove Theorem 5 (the bundled refinement property), we define the following equations with respect to any particular unisegment E.

$$\Psi(\text{MSIT}, \mathcal{E}, E, \mathcal{W})$$

$$= \mathcal{H}_1 \cdot \sum_{\text{E}' \in \text{MSIT} - \{E\}} w_{E'}$$

$$+ \mathcal{H}_2 \cdot \sum_{\text{E}', \text{E}'' \in \text{MSIT} - \{E\}, E \neq E''} F(E', E'') \cdot \frac{w_{E''}}{w_{E'}}$$

$$+ \mathcal{H}_3 \cdot \sum_{E', E'' \in \mathrm{MSIT}-\{E\}, E' \neq E''} F(E', E'') \cdot \frac{1}{w_{E'}}$$

$$+ \mathcal{H}_4 \cdot \sum_{E' \in \mathrm{MSIT}-\{E\}} G(E') \cdot \frac{1}{w_{E'}} + \mathcal{H}_5 \cdot \sum_{E' \in \mathrm{MSIT}-\{E\}} H(E') \cdot \frac{1}{w_{E'}} \tag{22}$$

$$\Phi(\mathrm{MSIT}, \mathcal{E}, E, \mathcal{W}) = \mathcal{H}_1 + \mathcal{H}_2 \cdot \sum_{E' \in \mathrm{MSIT}-\{E\}} F(E', E) \cdot \frac{1}{w_{E'}} \tag{23}$$

$$\Theta(\mathrm{MSIT}, \mathcal{E}, E, \mathcal{W} = \mathcal{H}_2 \cdot \sum_{E' \in \mathrm{MSIT}-\{E\}} F(E, E') \cdot w_{E'} + \mathcal{H}_3 \cdot \sum_{E' \in \mathrm{MSIT}-\{E\}} F(E, E') + \mathcal{H}_4 \cdot G(E) + \mathcal{H}_5 \cdot H(E). \tag{24}$$

Then, we can rewrite the objective function (7) as follows.

$$t(\mathrm{MSIT}, \mathcal{E}, \mathcal{W}) = \Psi(\mathrm{MSIT}, \mathcal{E}, E, \mathcal{W}) + \Phi(\mathrm{MSIT}, \mathcal{E}, E, \mathcal{W}) \cdot w_E + \Theta(\mathrm{MSIT}, \mathcal{E}, E, \mathcal{W}) \cdot \frac{1}{w_E}. \tag{25}$$

We show the following Lemma 8.

LEMMA 8. *Given an MSIT, a segment-division $\mathcal{E}$, and a wiresizing solution $\mathcal{W}$, for any particular unisegment E under $\mathcal{E}$, if we divide E into a sequence of unisegments $E_1, E_2, \ldots,$ and E_k, let each new unisegment inherit the wire width assignment of E, and denote the resulting segment-division and wiresizing solution $\mathcal{E}'$ and $\mathcal{W}'$, respectively, then the following relations hold for any unisegment E' other than E.*

$$\Psi(\mathrm{MSIT}, \mathcal{E}, E', \mathcal{W}) = \Psi(\mathrm{MSIT}, \mathcal{E}', E', \mathcal{W}')$$

$$\Phi(\mathrm{MSIT}, \mathcal{E}, E', \mathcal{W}) = \Phi(\mathrm{MSIT}, \mathcal{E}', E', \mathcal{W}')$$

$$\Theta(\mathrm{MSIT}, \mathcal{E}, E', \mathcal{W}) = \Theta(\mathrm{MSIT}, \mathcal{E}', E', \mathcal{W}').^{10}$$

PROOF. It is not difficult to verify that Lemma 8 is true if the following hold:

[10]In general, under the modeling method used in Cong and Leung [1995], Cong and Koh [1994], Cong and He [1995a], and Sapatnekar [1994], for the Elmore delay t^{ij} between source N_i and sink N_j, we have $t^{ij}(\mathcal{E}, \mathcal{W}) = t^{ij}(\mathcal{E}', \mathcal{W}')$. That is, the Elmore delay is independent of the segment-division when given the wiresizing solution.

$$F(E_1, E') = F(E_2, E') = \cdots = F(E, E')$$

$$F(E', E_1) = F(E', E_2) = \cdots = F(E', E)$$

$$G(E_1) = G(E_2) = \cdots = G(E)$$

$$H(E_1) = H(E_2) = \cdots = H(E).$$

Assume unisegment E is in segment S. There are two cases for unisegment E'.

Case 1. E' is also in the same segment S. According to Lemma 1, if E is left of E'.

$$F(E_1, E') = F(E_2, E') = \cdots = F(E, E') = F_l(S)$$

$$F(E', E_1) = F(E', E_2) = \cdots = F(E', E) = F_r(S);$$

if E is right of E',

$$F(E_1, E') = F(E_2, E') = \cdots = F(E, E') = F_r(S)$$

$$F(E', E_1) = F(E', E_2) = \cdots = F(E', E) = F_l(S).$$

Case 2. E' is in segment S' different from segment S. According to Lemma 2,

$$F(E_1, E') = F(E_2, E') = \cdots = F(E, E') = F(S, S')$$

$$F(E', E_1) = F(E', E_2) = \cdots = F(E', E) = F(S', S).$$

Again assuming unisegment E is in segment S, according to Lemma 3,

$$G(E_1) = G(E_2) = \cdots = G(E) = G(S)$$

$$H(E_1) = H(E_2) = \cdots = H(E) = H(S).$$

As a result, Lemma 8 holds. □

Recall the definition for the local refinement operation; according to Lemma 8 and Equation (25), we can conclude that the following Lemma 9 holds.

LEMMA 9. *When given the wiresizing solution $\mathcal{W}$ and any particular unisegment E, the local refinement result for E with respect to $\mathcal{W}$ is independent of the segment-division for unisegments other than E.*

We give the following proof for Theorem 5 (the bundled refinement property).

PROOF. For any particular unisegment E under the current segment-division $\mathcal{E}$ in segment S of an MSIT, it may be divided into k unisegments

under the finest segment-division $\mathscr{E}_F$. From left to right, let them be $E_l = E_{F1}, E_{F2}, E_{F3}, \ldots, E_{Fk} = E_r$.

Without loss of generality, we assume $F_l(S) \geq F_r(S)$. For a wiresizing solution $\mathscr{W}$ that dominates the optimal solution $\mathscr{W}^*$, let $\mathscr{W}^b$ be the wiresizing solution after performing an BRU operation of $\mathscr{W}$ on E under $\mathscr{E}$. Then, we have $w_l^b = w_{F2}^b = w_{F3}^b = \cdots = w_r^b$ according to the definition of the BRU operation. Meanwhile, in the optimal wiresizing solution $\mathscr{W}^*$, we have $w_l^* \geq w_{F2}^* \geq w_{F3}^* \geq \cdots \geq w_r^*$ according to the local monotone property.

According to Lemma 9, w_l^b is also the local refinement result for unisegment E_l under the finest segment-division $\mathscr{E}_F$. Therefore, $w_l^b \geq w_l^*$. As a result, $w_l^b = w_{F2}^b = w_{F3}^b = \cdots = w_r^b \geq w_l^* \geq w_{F2}^* \geq w_{F3}^* \geq \cdots \geq w_r^*$. Recall that the bundled refinement of unisegment E does not change the wire width in the wiresizing solution $\mathscr{W}$ (dominating $\mathscr{W}^*$) for any unisegment E' other than $E_l, E_{F2}, E_{F3}, \ldots, E_r$. Thus, $\mathscr{W}^b$ still dominates $\mathscr{W}^*$.

The BRL case can be proved in a similar way. □

Proof of Theorem 6

THEOREM 6. *The lower and upper bounds provided by BWSA are $\mathscr{E}_F$-tight.*

PROOF. Let $\mathscr{E}$ be the wiresizing segment-division after BWSA. For any unisegment E under $\mathscr{E}$, l_E is longer than *minLength* (the length for all unisegments under the finest segment-division $\mathscr{E}_F$) if and only if E is a *convergent* unisegment, whose bounds can not be tightened any more.

If E is *minLength* long, according to Lemma 9, its lower and upper bounds given by the bundled refinement operations are the same as those given by local refinement operations under $\mathscr{E}_F$. Thus, if the bundled refinement operations cannot tighten the lower and upper bounds, neither can the local refinement operations under $\mathscr{E}_F$.

In conclusion, Theorem 6 holds. □

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