

# QuickYield: An Efficient Global-Search Based Parametric Yield Estimation with Performance Constraints

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## ABSTRACT

With technology scaling down to 90nm and below, many yield-driven design and optimization methodologies have been proposed to cope with the prominent process variation and to increase the yield. A critical issue that affects the efficiency of those methods is to estimate the yield when given design parameters under variations. Existing methods either use Monte Carlo method in performance domain where thousands of simulations are required, or use local search in parameter domain where a number of simulations are required to characterize the point on the yield boundary defined by performance constraints. To improve efficiency, in this paper we propose QuickYield, a yield surface boundary determination by surface-point finding and global-search. Experiments on a number of different circuits show that for the same accuracy, QuickYield is up to 519X faster compared with the Monte Carlo approach, and up to 4.7X faster compared with YENSS, the fastest approach reported in literature.

**Categories and Subject Descriptors:** B.7.[Hardware]: - Integrated Circuits-Design Aids

**General Terms:** Algorithms, Performance

**Keywords:** Parametric yield, Circuit simulation

## 1. INTRODUCTION

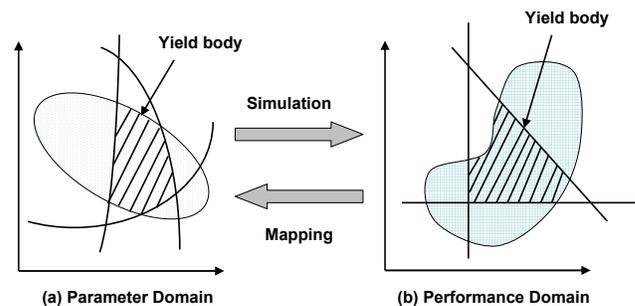
As integrated circuits enter into the nanometer era, process variation has become a major challenge for both design and fabrication [1, 2, 3, 4, 5, 6]. Circuit parameters such as effective channel length and threshold voltage of transistors can deviate significantly from their nominal values specified by designers, due to many uncertainties from the manufacturing process such as lithography, chemical mechanical polishing (CMP), etching and so on. The circuit performance such as delay and output swing may differ from the design specification under the nominal condition. Parametric yield, defined as the percentage of circuits that can work under a performance metric within the acceptable region, is a common measure to evaluate the design robustness in the presence of process variations.

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In general, the parametric yield can be estimated in either the performance domain or the parameter domain, as shown in Fig.(1). Monte Carlo simulation is commonly used for yield estimation in the performance domain. With the probability distribution of variable parameters, it first generates tens of thousands of random-sampling points in the parameter space, and then performs circuit simulations at each sampling point to evaluate the performance metric of interest. Within the obtained performance, Monte Carlo method screens out those missing the given performance constraints, and identify the acceptable or successful samplings. In this way, by dividing the number of successful samplings by that of the total sampling points, one can obtain the yield rate. The advantage of Monte Carlo simulation is its simplicity and generality. However, the Monte Carlo method is time-consuming. The lengthy simulation at each sampling point limits its application for the yield estimation and hence a fast analysis is required.



**Figure 1: Yield estimation in (a) parameter domain and (b) performance domain.**

Several works [1, 2, 3] have been proposed to utilize the *surface boundary*, which separates the success and failure regions to estimate the yield rate. These methods avoid the massive sampling simulations by only using points on the surface boundary. The approach in [1] uses linear constraints to approximate the yield surface. It has two limitations: first, the linear functions cannot approximate the yield body with a good accuracy, especially for yields defined by nonlinear performance constraints. Second, this method is based on the assumption that the yield body needs to be convex, which does not hold in general. To address these issues, approaches in [2] and [3] derive a nonlinear

procedure, called YENSS, which can handle nonlinear performance constraints as well as non-convex yield body. It starts from the nominal performance space, and searches along the tangent of the surface boundary to approach the surface boundary point, which is called *local search* in this paper. This method can provide the good accuracy without using the Monte Carlo method. However, expensive simulations are still required to locate each point on the surface boundary due to local searching.

In this paper, we propose an efficient parametric yield calculation method, called *QuickYield*. QuickYield applies a surface-point finding together with a global-search to locate points on the surface boundary. We first include performance constraints into the differential algebra equation (DAE) that describes the circuit and build an augmented equation system. The points on the surface boundary in the parameter domain can be determined by solving the augmented system equation. Then yield can be estimated from the obtained surface boundary. QuickYield is able to benefit problems where simulation is extremely expensive. Experimental results show that for the same accuracy, QuickYield is up to 519X faster compared with the Monte Carlo approach, and up to 4.7X faster compared with YENSS, the fastest approach reported in the literature.

The rest of the paper is organized in the following manner. We first review the background of the yield estimation in Section 2. In Section 3, we present the details about yield estimation in QuickYield. We present experimental results in Section 4 and conclude the paper in Section 5.

## 2. PRELIMINARIES

Generally, the variational parameters with performance constraint that defines the acceptable region can form a hyper-volume with respect to the nominal point in the parameter space, as shown in the Fig(2). The shaded region is the “safe” region where parameters can result in the acceptable performance. The parametric yield estimation, accordingly, is to calculate the ratio of the volume of the safe region to that of the entire space bounded by the min and max of all parameters. Here, all parameters are assumed independent after decomposing the correlation.

One direct approach to find the yield is to determine all the points in the hyper-volume. However, it is expensive and unnecessary, because the points on the hyper-surface boundary can provide enough information to calculate the volume of the yield body and thus the yield rate [3]. Therefore, it is desirable to directly identify the hyper-surface boundary (or called surface boundary).

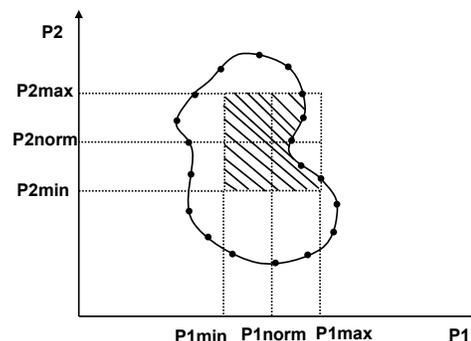
The hyper-surface that encloses the acceptable or safe region can be mathematically described by the performance constraint as <sup>1</sup>

$$H(\gamma_p; f_m) = f_m(\gamma_p) - f_{worst} = 0, \quad (1)$$

where  $\gamma_p$  denotes the parameters subject to the process variations. Depending on the specific circuit problem under study,  $\gamma_p$  can be different parameters, such as the threshold voltage, width, length, load capacitance and etc.  $f_m(\gamma_p)$  is the performance metric corresponding to  $\gamma_p$ , which can

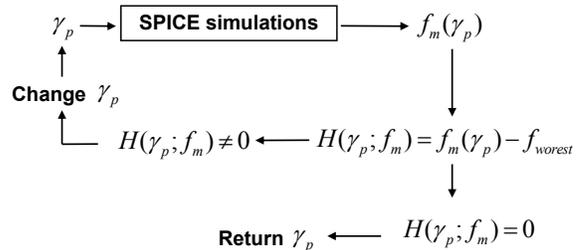
<sup>1</sup>For simplicity of our presentation and similar to [3], from now on we restrict our discussion to the simple case with only one performance constraint. It is understood, however, that our method can also be applied to multiple constraints scenario by handling one constraint at a time.

be swing, gate delay, oscillator frequency and so on. They can be obtained by SPICE simulations.  $f_{worst}$  is the worst-case performance that can be accepted. Unfortunately, in most cases it is either prohibitively expensive or technically impossible to obtain an explicit expression for  $f_m(\gamma_p)$ .

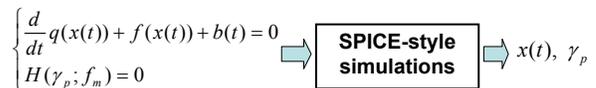


**Figure 2: Performance Constraints defines hyper-volume in parameter space**

There are some alternative ways proposed to obtain this surface. As shown in Fig.(3(a)), existing methods [1, 2, 3] start with an initial  $\gamma_p$ , and perform SPICE simulation with  $\gamma_p$  to get performance  $f_m(\gamma_p)$ . Then, the performance constraint  $H(\gamma_p; f_m)$  is checked with the obtained  $f_m$ . If  $\gamma_p$  cannot satisfy the constraints, a new  $\gamma_p$  is selected and the above procedure is repeated until the constraint is satisfied. In this way, the final calculated parameter lies on the surface boundary. However, those methods all require quite a number of simulations to identify points on the surface. The efficiency should be further improved.



(a) Existing methods



(b) QuickYield

**Figure 3: Comparison of boundary surface finding methods by existing methods and QuickYield**

## 3. YIELD ESTIMATION

In this section, we present details for the yield estimation in QuickYield. Without loss of generality, we assume that all the parameters are uniformly distributed. Non-uniform distribution and correlated parameters can be mapped into uniform parameter domain with pre-scale and PCA techniques[2, 3].

### 3.1 Algorithm Overview

In order to improve the efficiency when locating the yield surface boundary, the “surface-point finding” is developed to locate surface boundary points with aid of a *global search*, which is in contrast to the approach in YENSS [3]. YENSS performs the search of surface point mainly based on a *local search*, where the tangent direction is calculated during each step of searching. The determination of the tangent direction requires to calculate the sensitivity  $\partial f_m / \partial \bar{p}$ , which is expensive for each step of searching.

Our approach switches the role of performance constraint  $H(\gamma_p; f_m)$  and variable parameters  $\gamma_p$ , which is shown in Fig.3(b)). We treat the  $\gamma_p$  as an unknown, while introducing  $H(\gamma_p; f_m)$  as a extra equation into the simulation environment. The objective of the simulation is to find the value of  $\gamma_p$  that can satisfy the performance constraint  $H(\gamma_p; f_m)$  exactly. Therefore, the simulator needs to solve one augmented nonlinear system. With the obtained parameter values, the points on the surface boundary can be located in the parameter domain, and the parametric yield can be evaluated accordingly. Moreover, during the searching, we develop a strategy by the global-search, which avoids the expensive local search used in [3] yet with a high accuracy. In this way, we can develop an efficient yield estimation algorithm, called *QuickYield*. The flow is presented below.

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#### Algorithm 1 Flow of QuickYield

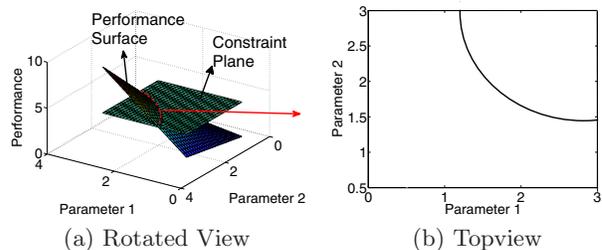
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- 1: /\* Setup \*/
  - 2: Input circuit constructs, nominal parameters and performance constraints;
  - 3: Build the augmented equation system with performance constraint equation;
  - 4:
  - 5: /\* Initial Yield Approximation \*/
  - 6: Find the intersection of surface boundary with each axis using Surface-point Finding Strategy.
  - 7: Initial yield approximation can be calculated with the yield volume defined by these intersections.
  - 8:
  - 9: **while** Yield estimation can be refined **do**
  - 10: /\* Surface-point Finding \*/
  - 11: Solve the augmented system to obtain the parameter values that fix an exact point on the surface boundary.
  - 12: /\* Yield Refinement \*/
  - 13: Refine the yield estimate from the additional points on the surface boundary.
  - 14: **end while**
- 

### 3.2 Surface-point Finding Strategy

For the purpose of illustration, we first consider two uncorrelated and uniform-distributed parameters  $p_1$  and  $p_2$  as shown in Fig.(2). Assuming that the performance constraint can be successfully added at one pair of nominal points  $(p_{1\_norm}, p_{2\_norm})$ , there exists a “safe” region inside the region bounded by  $(p_{1\_min}, p_{1\_max})$  and  $(p_{2\_min}, p_{2\_max})$ . The “safe” region thereby can be viewed as a nonlinear surface boundary separating the success and failure regions in the parameter domain. As such, the yield can be approximated by the area ratio of shaded region to the entire region bounded by min and max values of two parameters.

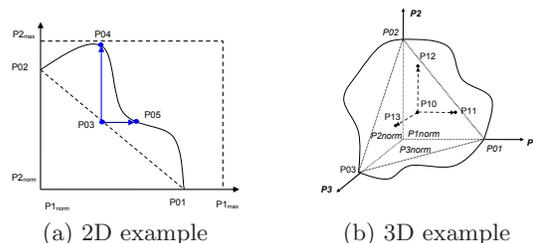
We elaborate the need to find surface points further from a geometrical perspective as shown in Fig.(4(a)). The equa-



**Figure 4: (a)rotated view and (b) topview of the intersection boundary of performance surface and constraint plane.**

tion system determines a performance surface in the parameter domain. The performance constraint  $H(\gamma_p; f_m)$  further locates a constraint plane  $f_m(\gamma_p) = f_{worst}$ <sup>2</sup>. It is clear that the yield surface boundary in the parameter domain, shown in Fig.(4(b)), is the intersection boundary of these two surfaces. Therefore, by solving the equation system together with the performance constraint, one can obtain their intersection boundary, which in fact separates the success and failure regions in the parameter domain.

For this sake, we can first locate boundary points on the nonlinear yield surface boundary as many as possible, and then the exact surface boundary can be approximated by connecting those identified surface points with the further refinement from the centroid point of those existing surface points. Such a surface-point finding strategy is called global-search in this paper, which is different from the local-search in YENSS by computing tangent direction at each point.

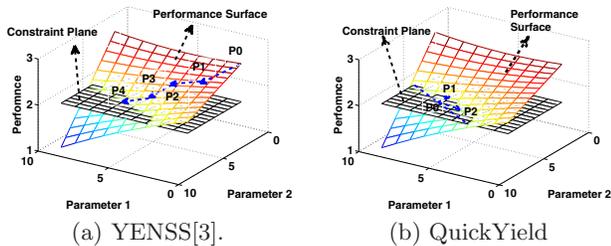


**Figure 5: Surface-point Finding Strategy**

Let’s illustrate the global-search by an example as follows. In order to find surface points, QuickYield first calculates the intersection points at each parameter axis:  $P_{01}$ ,  $P_{02}$  shown in Fig.(5(a)). Then, it fixes one coordinate of  $P_{01}$  or  $P_{02}$  at its nominal values, while introducing the other coordinate as the extra unknown  $\gamma_p$  as shown in Fig.(3(b)). By connecting these points, an initial approximation to the yield body can be generated. To have an accurate approximation of the yield body, additional surface points such as  $P_{04}$  and  $P_{05}$  in between can be further found by fixing one parameter of the middle point ( $P_{03}$ ) of  $(P_{01}, P_{02})$  and solving for the other parameter. This procedure is repeated to find more surface points to refine the yield estimation. Note that

<sup>2</sup>For the simplicity of illustration, a linear performance constraint is used in Fig.(4(a)). But QuickYield can handle fully nonlinear performance constraints in general.

this approach can be generalized to the case when there are multiple parameters. As shown in Fig.(5(b)), there exists a hyper-surface by connecting all intersections on axes, and hence the centroid point of the hyper-surface can be used to find additional surface points with the similar procedure in 2D problem.



**Figure 6: Comparison of Surface-point Finding Strategy**

It is worthwhile to further illustrate that our QuickYield uses the global-search, different from YENSS [3]. As shown in Fig.(6(b)),  $P_0$  is the initial middle point of the line connecting two axis intersection points. Obviously, its determination needs no simulation. The identified surface points, such as  $P_1$  and  $P_2$ , only require one simulation in QuickYield. Shown in Fig.(6(a)), YENSS involves more runs of simulations than QuickYield to locate a single surface point. This is because YENSS starts from the nominal design point and locally searches along the tangent direction of performance surface. The determination of the tangent direction requires to calculate the sensitivity  $\partial f_m / \partial \vec{p}$ , which is expensive. In contrast, the surface-point finding strategy in QuickYield method is more efficient since it searches in global fashion by starting from the middle of the identified points. The efficiency of QuickYield is also demonstrated in experiments.

### 3.3 Surface-point Finding Method

To determine the surface point numerically, we apply the parameter finding used for device optimization [7, 8] in the content of yield estimation, i.e., the surface-point finding.

We start from differential algebra equation (DAE) in (2)

$$\frac{d}{dt}q(x(t)) + f(x(t)) + b = 0 \quad (2)$$

where  $x(t)$  are unknown state variables.

First, QuickYield leverages the DAE with one performance constraint  $H(\gamma_p; f_m)$  in the augmented system equation (3). As such, we have a new system with  $x(t)$  and  $\gamma_p$  as the unknowns.

$$\begin{cases} \frac{d}{dt}q(x(t), \gamma_p) + f(x(t), \gamma_p) + b(t) = 0 \\ H(x(t), \gamma_p) = f_m(\gamma_p) - f_{worst} = 0 \end{cases} \quad (3)$$

We can use  $F(x(t), \gamma_p)$  to denote the left hand side of equation (3), and  $F: \mathbb{R}^m \times \mathbb{R}$  to rewrite the equations as  $F(x(t), \gamma_p) = 0$ . The system consists of a number of non-linear equations, and can be solved with Newton-Raphson iterations.

The Newton-Raphson iteration<sup>3</sup> requires the Jacobian matrix  $J(X) = \partial F / \partial X$ , where  $X = [x^T; \gamma_p]$  is the vector of the

<sup>3</sup>For the simplicity of illustration, from now on we omit the iteration index  $k$  in our discussion about Newton-Raphson iteration.

unknowns. For DC analysis, the Jacobian matrix becomes simple as:

$$J(X) = \begin{bmatrix} \partial f / \partial x & \partial f / \partial \gamma_p + b \\ \partial H(\gamma_p; x) / \partial x & \partial H(\gamma_p; x) / \partial \gamma_p \end{bmatrix} \quad (4)$$

Note that for the periodical circuits such as ring oscillators, the periodical steady state analysis (PSS) is required to find their periods  $T$  [9, 7, 8]. Similarly, one augmented system for PSS analysis can be built in discrete time domain with the finite difference method as [8, 7]:

$$\begin{bmatrix} \frac{1}{h}(q_1 - q_n) + f_1 + b \\ \frac{1}{h}(q_2 - q_1) + f_2 + b \\ \vdots \\ \frac{1}{h}(q_n - q_{n-1}) + f_n + b \\ H(x^T; \gamma_p) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix} \quad (5)$$

Accordingly, the Jacobian matrix  $J_{fd}$  can be expressed as equation (6) at the top of the next page, where  $C = \partial q / \partial x$ , and  $G = \partial f / \partial x$ . When Newton-Raphson iteration converges, it returns the state-variable  $x(t)$  and the variational parameter  $\gamma_p$  which resides on the boundary of the yield body in the parameter domain.

### 3.4 Surface-point Based Yield Estimation

To estimate the yield rate, we first employ an analytical formula [3] to compute the volume of the parallelepiped and the simplex defined by the intersection points of the boundary surface with each parameter axis. We then use it as the initial approximation to the yield. When additional points on the yield boundary identified, QuickYield will refine the yield volume approximation by adding or subtracting the hyper-volume increments. Accordingly, the yield can be simply obtained if one divides the calculated volume by that of the parameter space.

## 4. EXPERIMENTAL RESULTS

We have implemented QuickYield in a Matlab-based circuit simulator, and all experiments are carried out on a Linux server with a 2.4GHz Xeon processor and 4GB memory. We use a Schmitt Trigger and a three-stage ring oscillator to compare the accuracy and efficiency of QuickYield with Monte Carlo and YENSS [3]. As an illustration, we use the widths of the MOSFETs as parameters  $\gamma_p$  subject to process variation, but QuickYield can handle other variations, such as threshold voltage, load capacitance and channel length as well.

### 4.1 Schmitt Trigger

We first use a Schmitt Trigger (shown in Fig.(7)) to verify the accuracy and efficiency of QuickYield combined with DC analysis. In this case, we consider the lower switching threshold  $V_{TL}$  as the performance metric. We consider the channel widths of NMOS  $W_{n1}$  and PMOS  $W_{p2}$  as variational parameters  $\gamma_p$ , which have 30% variations from their nominal values. As such,  $V_{TL}$  can differ from nominal value, which change the lower switching threshold  $V_{TL}$ . The performance constraint is given as: when the input  $V_{TL}$  is at 0.4V and the output is initially set to Vdd, the output  $V_{OHH}$  should be greater than 1.7V.

First, 30% random variations are introduced to  $W_{p2}$  and  $W_{n1}$  separately. The result from 6000 Monte Carlo DC simulations is shown as Fig.(8(b)) in the parameter space



We further validate QuickYield against Monte Carlo simulation as shown in Fig.(10(a)). There are also two curves: upper bound for  $T_{min}$  and low bound for  $T_{max}$ , and they are identical to the result from Monte Carlo in Fig.(10(b)).

Moreover, we compare the accuracy and runtime of Monte-Carlo, YENSS obtained from [3] by normalizing with respect to its Monte Carlo runtime, and QuickYield in Table.(2). From this table, the Monte Carlo with 5000 simulations can generate a more accurate result, while it needs much more computation time. According to [3], YENSS can achieve 3.4% accuracy with 139X speedup over Monte Carlo. But QuickYield can obtain 519X speedup over Monte Carlo at a similar accuracy. Note that QuickYield and YENSS only use 10 points to locate two boundary curves. The accuracy can be improved with additional points.

We also study how the accuracy and runtime of QuickYield scales with the number of points, and the results is shown in Fig. 11. From the figure we can see that the runtime increases linearly, while the yield quickly converges when the point is over 60.

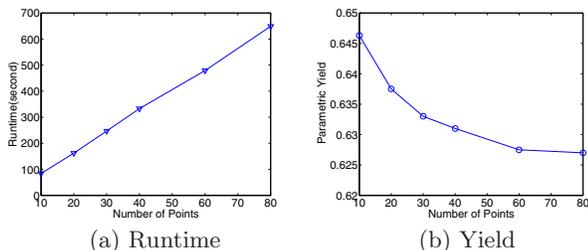


Figure 11: Runtime and accuracy scalability w.r.t. the number of points.

Table 2: Accuracy and Time Comparison for Ring Oscillator with two variational parameters

method	yield	time (second)	speedup
Monte Carlo (1000)	0.67465	8514.7	5.17x
Monte Carlo (5000)	0.62658	44073.8	1x
YENSS (10 points)	0.6482	-	139X
QuickYield (10 points)	0.6463	84.9	519x

To demonstrate the ability to handle many variational parameters, we increase the complexity of the problem by introducing random variation to the load capacitance  $C1$  in the first stage. First, Monte Carlo samples the 3-D parameter space, and simulation results are plotted in Fig.(12(b)) (blue domain denotes the success region). Next, QuickYield is applied to find the boundary surfaces separating the success/fail regions shown in Fig.(12(a)), which is identical to the hyper-volume from Monte Carlo. Note that we remove the front and back surfaces to exhibit the surfaces for  $T_{min}$  and  $T_{max}$ .

Similarly, the accuracy and time are summarized in Table(3). With 20 points on each boundary surface, QuickYield can not only achieve up to 0.6% accuracy, but also obtain 267X speedup over Monte Carlo and 4.6X over YENSS.

## 5. CONCLUSIONS AND FUTURE WORK

In this paper, we have proposed a fast algorithm, QuickYield, to calculate the parametric yield with the perfor-

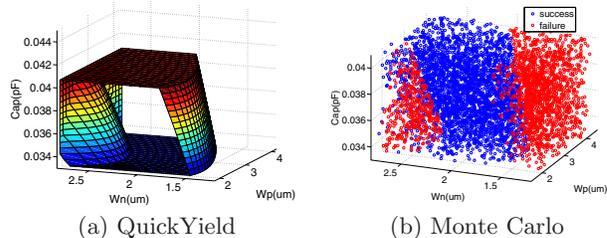


Figure 12: Simulation results from QuickYield and Monte Carlo on Ring Oscillator

Table 3: Accuracy and Time Comparison for Ring Oscillator with three variational parameters

method	yield	time (second)	speedup
Monte Carlo (1000)	0.648	11840	5.33X
Monte Carlo (5000)	0.617	63128	1X
YENSS (20 points)	0.623	-	57X
QuickYield (20 points)	0.621	236.9	267X

mance constraint. QuickYield leverages the DAE equation with the performance constraint together to build an augmented system. By solving it, QuickYield locates the yield boundary points in the parameter space with the global-search, and further calculates the yield rate efficiently. Experimental results show that for the same accuracy, QuickYield is up to 519X faster compared with the Monte Carlo approach, and up to 4.7X faster compared with YENSS [2, 3]. Future work is to extend QuickYield in general to handle more variables under a problem formulation in the high-dimensional parameter space.

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