Non-Linear Statistical Static Timing Analysis for Non-Gaussian Variation Sources

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[*]Dr. Xiong's work was finished while he was with UCLA

Outline

- Background and motivation
- Delay modeling
- Atomic operations for SSTA
- Experimental results
- Conclusions and future work

Motivation

- Gaussian variation sources ← not all variation is Gaussian in reality
  - Linear delay model, tightness probability [C.V DAC'04]
  - Quadratic delay model, tightness probability [L.Z DAC'05]
  - Quadratic delay model, moment matching [Y.Z DAC'05]
- Non-Gaussian variation sources ← computationally inefficient
  - Non-linear delay model, tightness probability [C.V DAC'05]
  - Linear delay model, ICA and moment matching [J.S DAC'06]
- Need fast and accurate SSTA for Non-linear Delay model with Non-Gaussian variation sources

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Delay Modeling

- Delay with variation

$$D = F(X_1, X_2, \ldots, X_i, \ldots)$$

- Linear delay model

$$D \approx d_0 + \sum a_i X_i$$

- Quadratic delay model

$$D = d_0 + \sum (a_i X_i + b_i X_i^2) + a_r X_r + b_r X_r^2$$

  - X_is are independent random variables with arbitrary distribution
    - Gaussian or non-Gaussian

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  - Max operation
  - Add operation
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Max Operation

- Problem formulation:
  - Given

$$D_1 = d_{01} + \sum (a_{i1} X_i + b_{i1} X_i^2) + a_{r1} X_{r1} + b_{r1} X_{r1}^2$$

$$D_2 = d_{02} + \sum (a_{i2} X_i + b_{i2} X_i^2) + a_{r2} X_{r2} + b_{r2} X_{r2}^2$$

  - Compute:

$$D = max(D_1, D_2)$$

Reconstruct Using Moment Matching

- To represent D=max(D1,D2) back to the quadratic form

$$D = \max(D_1, D_2) = d_0 + \sum_i (a_i X_i + b_i X_i^2) + a_r X_r + b_r X_r^2$$

- We can show the following equations hold

$$E[X_i \max(D_1, D_2)] = a_i m_{i,2} + b_i m_{i,3}$$

$$E[X_i^2 \max(D_1, D_2)] = E[\max(D_1, D_2)] m_{i,2} + a_i m_{i,3} + b_i (m_{i,4} - m_{i,2}^2)$$

  - $m_{i,k}$ is the kth moment of X_i, which is known from the process characterization
  - From the joint moments between D and X_is →the coefficients a_is and b_is can be computed by solving the above linear equations
- Use random term and constant term to match the first three moments of max(D_1, D_2)

Basic Idea

- Compute the joint PDF of D_1 and D_2
- Compute the moments of $\max(D_1,D_2)$
- Compute the Joint moments of Xi and $\max(D_1,D_2)$
- Reconstruct the quadratic form of $\max(D_1,D_2)$
  - Keep the exact correlation between $\max(D_1,D_2)$ and X_i
  - Keep the exact first-three moments of $\max(D_1,D_2)$

JPDF by Fourier Series

- Assume that D_1 and D_2 are within the ±3σ range
  - The joint PDF of D_1 and D_2, $f(v_1, v_2) \approx 0$, when v_1 and v_2 is not in the ±3σ range
- Approximate the Joint PDF of D_1 and D_2 by the first K_{th} order Fourier Series within the ±3σ range:

$$f(v_1, v_2) \approx \sum_{p,q=-K}^{K} \alpha_{pq} \cdot e^{\zeta_p v_1 + \eta_q v_2}$$

where $\zeta_p = jp\pi/l$ and $\eta_q = jq\pi/h$ with $j = \sqrt{-1}$

$l = 3\sigma_{D1}$, $h = 3\sigma_{D2}$

α_{ij} are Fourier coefficients

Fourier Coefficients

- The Fourier coefficients can be computed as:

$$\alpha_{pq} = \frac{1}{4lh}\int_{-l}^{l}\int_{-h}^{h} e^{-\zeta_p v_1 - \eta_q v_2} \cdot f(v_1, v_2) dv_1 dv_2$$

- Considering $f(v_1, v_2) \approx 0$ outside the range of $[-l, l; -h, h]$

$$\alpha_{pq} \approx \frac{1}{4lh} E[e^{-\zeta_p \Delta D_1 - \eta_q \Delta D_2}]$$

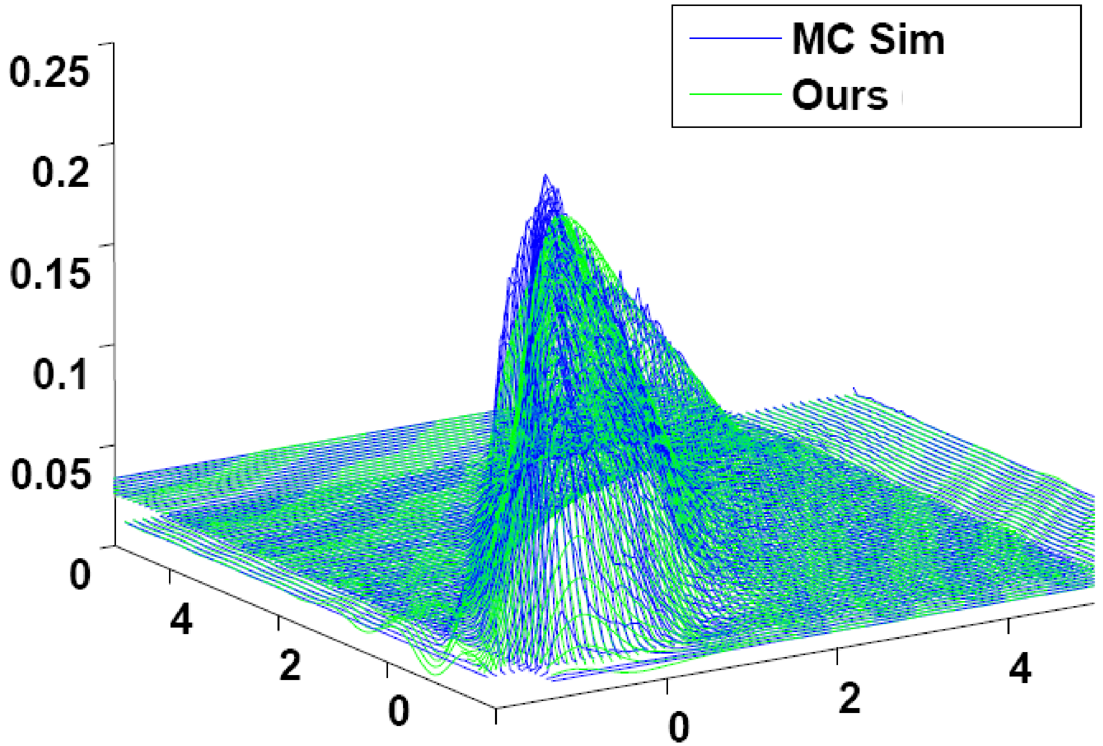
$$= \frac{1}{4lh} e^{-Y_{c,pq}} E[e^{-Y_{r1,pq}}] E[e^{-Y_{r2,pq}}] \prod E[e^{-Y_{i,pq}}]$$

where

$$Y_{c,pq} = \zeta_p(d_{01} - \mu_1) + \eta_q(d_{02} - \mu_2)$$
$$Y_{i,pq} = (\zeta_p a_{i1} + \eta_q a_{i2})X_i + (\zeta_p b_{i1} + \eta_q b_{i2})X_i^2$$

- $Y_{i,pq}$ can be written in the form of $Y = c_1 X_i + c_2 X_i^2$.
  - $E[e^{-Y_{i,pq}}]$ can be pre-computed and store in a 2-dimensional look up table indexed by c_1 and c_2

JPDF Comparison



- Assume that all the variation sources have uniform distributions within [-0.5, 0.5]
  - Our method can be applies to arbitrary variation distributions
- Maximum order of Fourier Series K=4

Moments of D=max(D_1, D_2)

- The t_{th} order raw moment of D=max(D_1,D_2) is

$$M_t = \iint_{v_1 > v_2} v_1^t f(v_1, v_2) dv_1 dv_2 + \iint_{v_2 > v_1} v_2^t f(v_1, v_2)) dv_1 dv_2$$

- Replacing the joint PDF with its Fourier Series:

$$M_t = \sum_{p,q=-k}^{k} \alpha_{pq} \cdot L(t, p, q, l, h, \mu_1, \mu_2)$$

where

$$L = \iint_{v_1 > v_2} v_1^t e^{\zeta_p (v_1 - \mu_1) + \eta_q (v_2 - \mu_2)} dv_1 dv_2 + \iint_{v_2 > v_1} v_2^t e^{\zeta_p (v_1 - \mu_1) + \eta_q (v_2 - \mu_2)} dv_1 dv_2$$

  - L can be computed using close form formulas

- The central moments of D can be computed from the raw moments

Joint Moments

- Approximate the Joint PDF of X_i, D_1, and D_2 with Fourier Series:

$$f(x_i, v_1, v_2) \approx \sum_{p,q,s=-K}^{K} \beta^i_{pqs} \cdot e^{\xi_{i,s} x_i + \zeta_p v_1 + \eta_q v_2}$$

  - The Fourier coefficients β^i_{pqs} can be computed in the similar way as α_{pq}

- The joint moments between D and X_is are computed as:

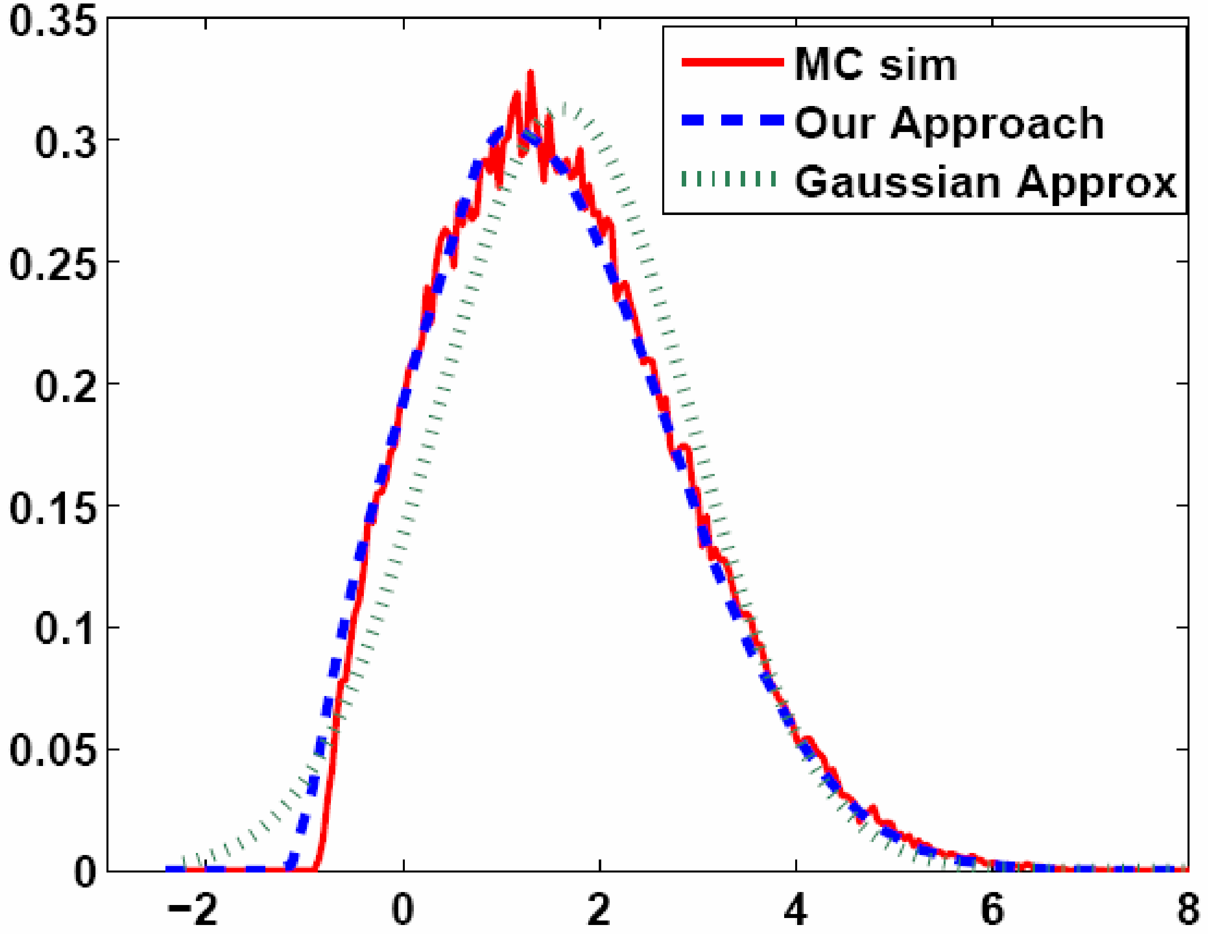
$$\begin{aligned} Ec_{i,t} &= E[X_i^t \cdot max(D_1, D_2)] \\ &= \iiint_{v_1 > v_2} x_i^t v_1 f(x_i, v_1 - \mu_1, v_2 - \mu_2) dx_i dv_1 dv_2 + \\ &\quad \iiint_{v_2 > v_1} x_i^t v_2 f(x_i, v_1 - \mu_1, v_2 - \mu_2) dx_i dv_1 dv_2. \end{aligned}$$

  - Replacing the *f* with the Fourier Series

$$Ec_{i,t} = \sum_{p,q,s=-K}^{K} \beta^i_{pqs} J(t, \xi_{i,s}, -w_i, w_i) L(1, p, q, l, h, \mu_1, \mu_2)$$

where $J(t, \gamma, \tau_1, \tau_2) = \int_{\tau_1}^{\tau_2} x^t e^{\gamma x}$

PDF Comparison for One Step Max



- Assume that all the variation sources have uniform distributions within [-0.5, 0.5]

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Add Operation

- Problem formulation
  - Given D_1 and D_2, compute $D=D_1+D_2$
- Just add the correspondent parameters to get the parameters of D

$$d_0 = d_{01} + d_{02}$$
$$a_i = a_{i1} + a_{i2}$$
$$b_i = b_{i1} + b_{i2}$$

- The random terms are computed to match the second and third order moments of D

Complexity Analysis

- Max operation
  - $O(nK^3)$

  Where n is the number of variation sources and K is the max order of Fourier Series
- Add operation
  - $O(n)$
- Whole SSTA process
  - The number of max and add operations are linear related to the circuit size

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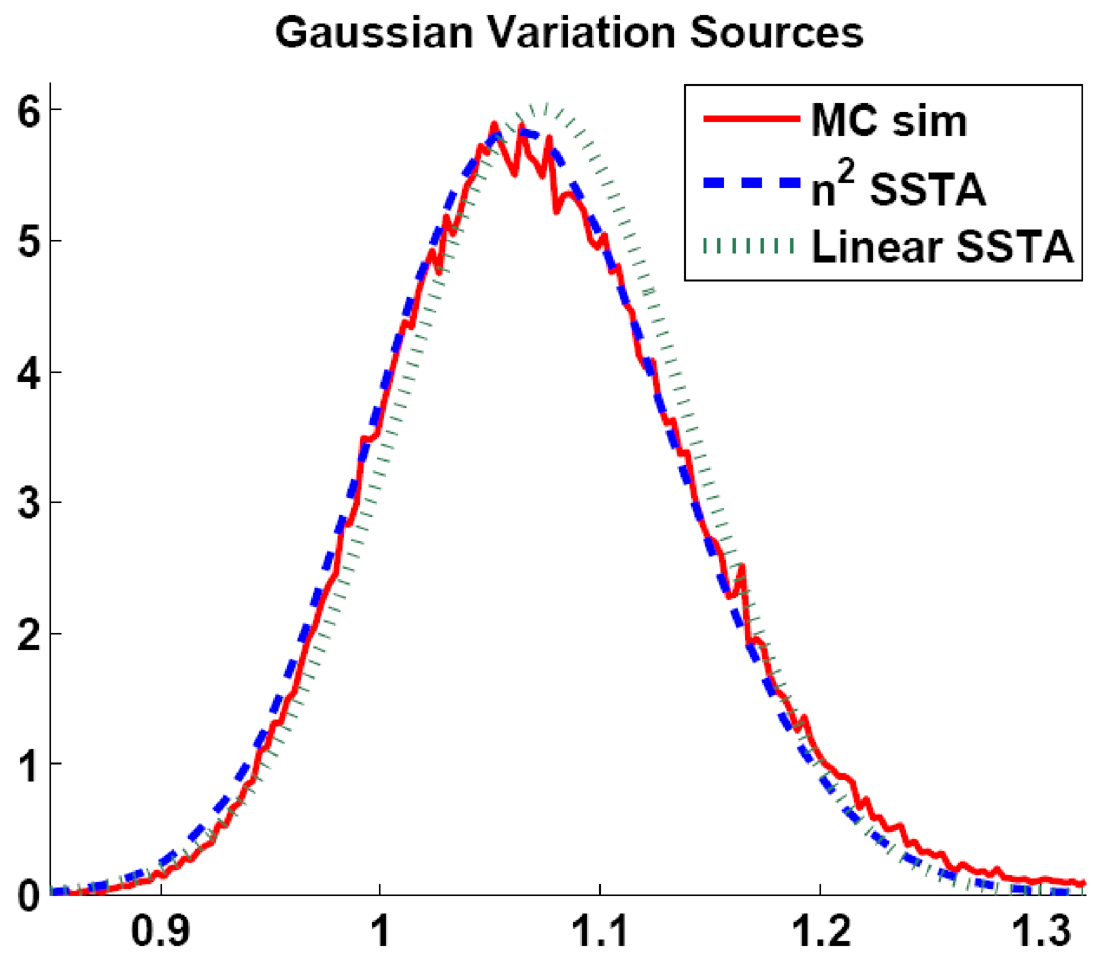
Experimental Setting

- Variation sources:
  - Gaussian only
  - Non-Gaussian
    - Uniform
    - Triangle
- Comparison cases
  - Linear SSTA with Gaussian variation sources only
    - Our implementation of [C.V DAC04]
  - Monte Carlo with 100000 samples
- Benchmark
  - ISCAS89 with randomly generated variation sensitivity

PDF Comparison



- PDF comparison for s5738
- Assume all variation sources are Gaussian

Mean and Variance Comparison for Gaussian Variation Sources

Bench mark	Monte Carlo		n^2SSTA		linSSTA	
	σ/μ %	95%	σ/μ %	95%	σ/μ %	95%
s27	15.9	1.50	14.9	1.48	13.9	1.47
s386	15.7	1.50	14.9	1.48	14.1	1.46
s444	15.7	1.49	14.9	1.47	14.2	1.46
s832	15.7	1.49	14.8	1.46	14.1	1.45
s1494	16.1	1.50	15.5	1.47	14.4	1.46
s5378	15.8	1.48	14.6	1.46	14.0	1.46
Avg Error	-	-	5.5%	1.61%	10.9%	1.88%

Mean and Variance Comparison for non-Gaussian Variation Sources

Bench mark	Monte Carlo			n^2SSTA		
	σ/μ %	95% yield	run time (s)	σ/μ %	95% yield	run time (s)
Uniform Variation Sources						
s27	14.7	1.41	3.4	14.8	1.41	0.80
s386	14.9	1.41	61	14.9	1.41	2.00
s444	15.1	1.42	44	14.8	1.42	3.07
s832	15.0	1.41	91	14.5	1.41	5.24
s1494	15.4	1.41	285	15.6	1.41	7.97
s5378	15.3	1.42	855	14.9	1.42	27.1
Avg	-	-	-	1.37%	0.01%	1/22.3
Tri-angle Variation Sources						
s27	13.6	1.44	4.3	13.8	1.44	0.80
s386	13.6	1.45	61	13.7	1.45	1.88
s444	14.2	1.47	57	14.3	1.47	2.99
s832	15.0	1.48	115	15.0	1.48	6.81
s1494	14.1	1.45	284	14.3	1.45	7.60
s5378	13.9	1.45	903	14.0	1.45	25.6
Avg	-	-	-	0.73%	0.01%	1/24.4

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Conclusion and Future Work

- We propose a novel SSTA technique is presented to handle both non-linear delay dependency and non-Gaussian variation sources
- The SSTA process are based on look up tables and close form formulas
- Our approach predict all timing characteristics of circuit delay with less than 2% error
- In the future, we will move on to consider the cross terms of the quadratic delay model

Thank you