Non-Linear Statistical Static Timing Analysis for Non-Gaussian Variation Sources

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\textsuperscript{*}Dr. Xiong's work was finished while he was with UCLA
Outline

- Background and motivation
- Delay modeling
- Atomic operations for SSTA
- Experimental results
- Conclusions and future work
Motivation

- Gaussian variation sources $\Leftarrow$ not all variation is Gaussian in reality
  - Linear delay model, tightness probability [C.V DAC’04]
  - Quadratic delay model, tightness probability [L.Z DAC’05]
  - Quadratic delay model, moment matching [Y.Z DAC’05]

- Non-Gaussian variation sources $\Leftarrow$ computationally inefficient
  - Non-linear delay model, tightness probability [C.V DAC’05]
  - Linear delay model, ICA and moment matching [J.S DAC’06]

- Need fast and accurate SSTA for Non-linear Delay model with Non-Gaussian variation sources
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Delay Modeling

- Delay with variation
  \[ D = F(X_1, X_2, \ldots, X_i, \ldots) \]

- Linear delay model
  \[ D \approx d_0 + \sum a_i X_i \]

- Quadratic delay model
  \[ D = d_0 + \sum (a_i X_i + b_i X_i^2) + a_r X_r + b_r X_r^2 \]

- \( X_i \)s are independent random variables with arbitrary distribution
  - Gaussian or non-Gaussian
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  - Max operation
  - Add operation
  - Complexity analysis
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Max Operation

Problem formulation:

Given

\[ D_1 = d_{01} + \sum (a_{i1}X_i + b_{i1}X_i^2) + a_{r1}X_{r1} + b_{r1}X_{r1}^2 \]

\[ D_2 = d_{02} + \sum (a_{i2}X_i + b_{i2}X_i^2) + a_{r2}X_{r2} + b_{r2}X_{r2}^2 \]

Compute:

\[ D = max(D_1, D_2) \]
Reconstruct Using Moment Matching

- To represent $D = \max(D_1, D_2)$ back to the quadratic form

$$D = \max(D_1, D_2) = d_0 + \sum_i (a_i X_i + b_i X_i^2) + a_r X_r + b_r X_r^2$$

- We can show the following equations hold

$$E[X_i \max(D_1, D_2)] = a_i m_{i,2} + b_i m_{i,3}$$

$$E[X_i^2 \max(D_1, D_2)] = E[\max(D_1, D_2)] m_{i,2} + a_i m_{i,3} + b_i (m_{i,4} - m_{i,2}^2)$$

- $m_{i,k}$ is the $k$th moment of $X_i$, which is known from the process characterization
- From the joint moments between $D$ and $X_i$s → the coefficients $a_i$s and $b_i$s can be computed by solving the above linear equations

- Use random term and constant term to match the first three moments of $\max(D_1, D_2)$
Basic Idea

- Compute the joint PDF of $D_1$ and $D_2$
- Compute the moments of $\max(D_1, D_2)$
- Compute the Joint moments of $X_i$ and $\max(D_1, D_2)$
- Reconstruct the quadratic form of $\max(D_1, D_2)$
  - Keep the exact correlation between $\max(D_1, D_2)$ and $X_i$
  - Keep the exact first-three moments of $\max(D_1, D_2)$
JPDF by Fourier Series

- Assume that $D_1$ and $D_2$ are within the $\pm 3\sigma$ range
  - The joint PDF of $D_1$ and $D_2$, $f(v_1, v_2) \approx 0$, when $v_1$ and $v_2$ is not in the $\pm 3\sigma$ range

- Approximate the Joint PDF of $D_1$ and $D_2$ by the first $K_{th}$ order Fourier Series within the $\pm 3\sigma$ range:

$$f(v_1, v_2) \approx \sum_{p, q=-K}^{K} \alpha_{pq} \cdot e^{\zeta_p v_1 + \eta_q v_2}$$

where $\zeta_p = j p \pi / l$ and $\eta_q = j q \pi / h$ with $j = \sqrt{-1}$

$l = 3\sigma_{D_1}$, $h = 3\sigma_{D_2}$

$\alpha_{ij}$ are Fourier coefficients
The Fourier coefficients can be computed as:

\[
\alpha_{pq} = \frac{1}{4lh} \int_{-l}^{l} \int_{-h}^{h} e^{-\zeta_p v_1 - \eta_q v_2} \cdot f(v_1, v_2) dv_1 dv_2
\]

- Considering \( f(v_1, v_2) \approx 0 \) outside the range of \([-l, l; -h, h]\)

\[
\alpha_{pq} \approx \frac{1}{4lh} E[e^{-\zeta_p \Delta D_1 - \eta_q \Delta D_2}]
\]

\[
= \frac{1}{4lh} e^{-Y_{c,pq}} E[e^{-Y_{r1,pq}}] E[e^{-Y_{r2,pq}}] \prod E[e^{-Y_{i,pq}}]
\]

where

\[
Y_{c,pq} = \zeta_p (d_{01} - \mu_1) + \eta_q (d_{02} - \mu_2)
\]

\[
Y_{i,pq} = (\zeta_p a_{i1} + \eta_q a_{i2}) X_i + (\zeta_p b_{i1} + \eta_q b_{i2}) X_i^2
\]

- \( Y_{i,pq} \) can be written in the form of \( Y = c_1 X_i + c_2 X_i^2 \).
- \( E[e^{-Y_{i,pq}}] \) can be pre-computed and stored in a 2-dimensional look up table indexed by \( c_1 \) and \( c_2 \).
Assume that all the variation sources have uniform distributions within [-0.5, 0.5]
- Our method can be applied to arbitrary variation distributions

- Maximum order of Fourier Series K=4
Moments of $D=\max(D_1, D_2)$

- The $t_{th}$ order raw moment of $D=\max(D_1, D_2)$ is

$$M_t = \iiint_{v_1 > v_2} v_1^t f(v_1, v_2) dv_1 dv_2 + \iiint_{v_2 > v_1} v_2^t f(v_1, v_2) dv_1 dv_2$$

- Replacing the joint PDF with its Fourier Series:

$$M_t = \sum_{p, q = -k}^k \alpha_{pq} \cdot L(t, p, q, l, h, \mu_1, \mu_2)$$

where

$$L = \iiint_{v_1 > v_2} v_1^t e^{\zeta_p(v_1 - \mu_1) + \eta_q(v_2 - \mu_2)} dv_1 dv_2 +$$

$$\iiint_{v_2 > v_1} v_2^t e^{\zeta_p(v_1 - \mu_1) + \eta_q(v_2 - \mu_2)} dv_1 dv_2$$

- $L$ can be computed using close form formulas

- The central moments of $D$ can be computed from the raw moments
Joint Moments

- Approximate the Joint PDF of $X_i$, $D_1$, and $D_2$ with Fourier Series:

  \[ f(x_i, v_1, v_2) \approx \sum_{p, q, s = -K}^{K} \beta_{pq}^i \cdot e^{\xi_i, s x_i + \zeta_p v_1 + \eta_q v_2} \]

  - The Fourier coefficients $\beta_{pq}^i$ can be computed in the similar way as $\alpha_{pq}$

- The joint moments between $D$ and $X_i$s are computed as:

  \[
  Ec_{i,t} = E[X_i^t \cdot \max(D_1, D_2)]
  = \int \int \int_{v_1 > v_2} x_i^t v_1 f(x_i, v_1 - \mu_1, v_2 - \mu_2) dx_i dv_1 dv_2 + \\
  \int \int \int_{v_2 > v_1} x_i^t v_2 f(x_i, v_1 - \mu_1, v_2 - \mu_2) dx_i dv_1 dv_2.
  \]

  - Replacing the $f$ with the Fourier Series

  \[
  Ec_{i,t} = \sum_{p, q, s = -K}^{K} \beta_{pq}^i \int_{-w_i, w_i} J(t, \xi_i, s, -w_i, w_i) L(1, p, q, l, h, \mu_1, \mu_2)
  \]

  where

  \[
  J(t, \gamma, \tau_1, \tau_2) = \int_{\tau_1}^{\tau_2} x^t e^{\gamma x} \]
Assume that all the variation sources have uniform distributions within [-0.5, 0.5]
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  - Complexity analysis
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Add Operation

- Problem formulation
  - Given $D_1$ and $D_2$, compute $D = D_1 + D_2$

- Just add the correspondent parameters to get the parameters of $D$

\[
\begin{align*}
  d_0 &= d_{01} + d_{02} \\
  a_i &= a_{i1} + a_{i2} \\
  b_i &= b_{i1} + b_{i2}
\end{align*}
\]

- The random terms are computed to match the second and third order moments of $D$
Complexity Analysis

- Max operation
  - $O(nK^3)$
    - Where $n$ is the number of variation sources and $K$ is the max order of Fourier Series

- Add operation
  - $O(n)$

- Whole SSTA process
  - The number of max and add operations are linear related to the circuit size
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Experimental Setting

- **Variation sources:**
  - Gaussian only
  - Non-Gaussian
    - Uniform
    - Triangle

- **Comparison cases**
  - Linear SSTA with Gaussian variation sources only
    - Our implementation of [C.V DAC04]
  - Monte Carlo with 100000 samples

- **Benchmark**
  - ISCAS89 with randomly generated variation sensitivity
PDF Comparison

Gaussian Variation Sources

- PDF comparison for s5738
- Assume all variation sources are Gaussian
# Mean and Variance Comparison for Gaussian Variation Sources

<table>
<thead>
<tr>
<th>Bench Mark</th>
<th>Monte Carlo</th>
<th>$n^2$SSTA</th>
<th>linSSTA</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\sigma/\mu$ %</td>
<td>95%</td>
<td>$\sigma/\mu$ %</td>
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<tr>
<td>s27</td>
<td>15.9</td>
<td>1.50</td>
<td>14.9</td>
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<td>s386</td>
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<td>1.50</td>
<td>14.9</td>
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<td>s5378</td>
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<tr>
<td>Avg Error</td>
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<td>-</td>
<td>5.5%</td>
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</table>

The table above compares the mean and variance for Gaussian variation sources using Monte Carlo, $n^2$SSTA, and linSSTA methods. The average error across all benchmarks is highlighted in red.
Mean and Variance Comparison for non-Gaussian Variation Sources

<table>
<thead>
<tr>
<th>Bench mark</th>
<th>Monto Carlo</th>
<th>$n^2$SSTA</th>
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</thead>
<tbody>
<tr>
<td>$\sigma/\mu$</td>
<td>95% yield</td>
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<tr>
<td>Avg</td>
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We propose a novel SSTA technique is presented to handle both non-linear delay dependency and non-Gaussian variation sources.

The SSTA process are based on look up tables and close form formulas.

Our approach predict all timing characteristics of circuit delay with less than 2% error.

In the future, we will move on to consider the cross terms of the quadratic delay model.
Thank you