Incremental Latin Hypercube Sampling

for Lifetime Stochastic Behavioral Modeling of Analog Circuits

Yen-Lung Chen[+], Wei Wu[*], Chien-Nan Jimmy Liu[+] and Lei He[*]
EE Dept., National Central University, Taiwan[+]
EE Dept., University of California, Los Angeles, CA, USA[*]

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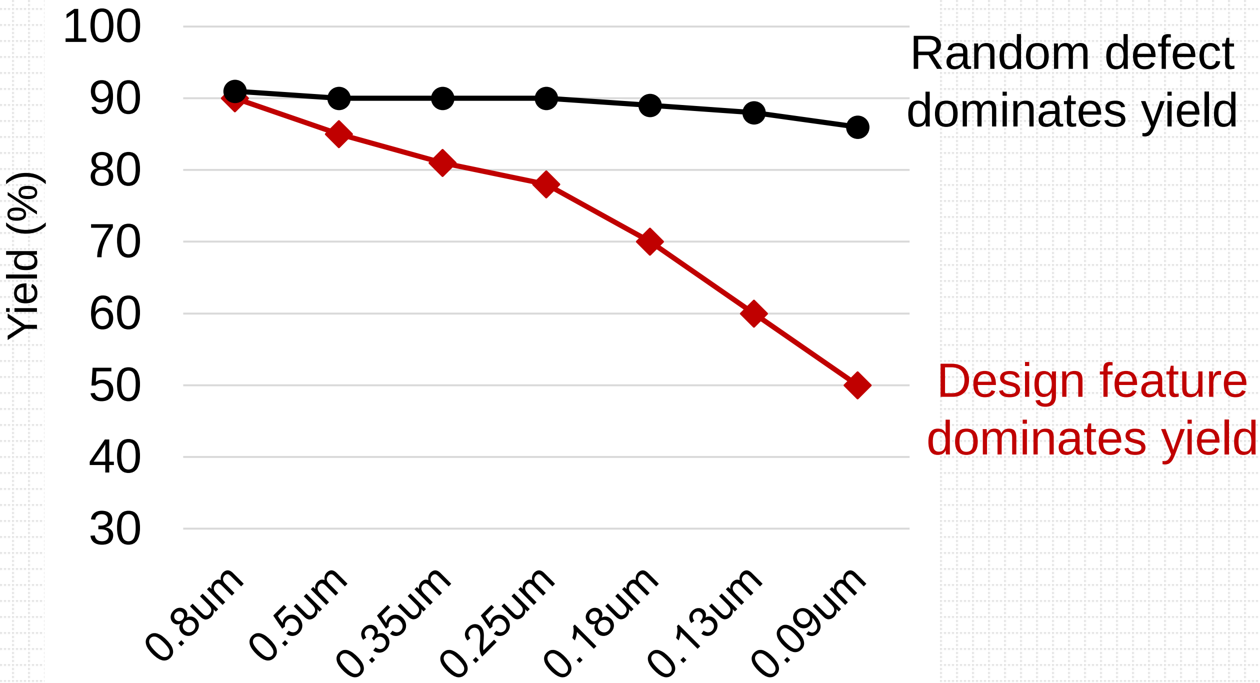
Outline

- Background of Lifetime Yield Analysis
- Proposed Incremental Latin Hypercube Sampling
- Experimental Results
- Conclusions

Process Variations & Yield Loss

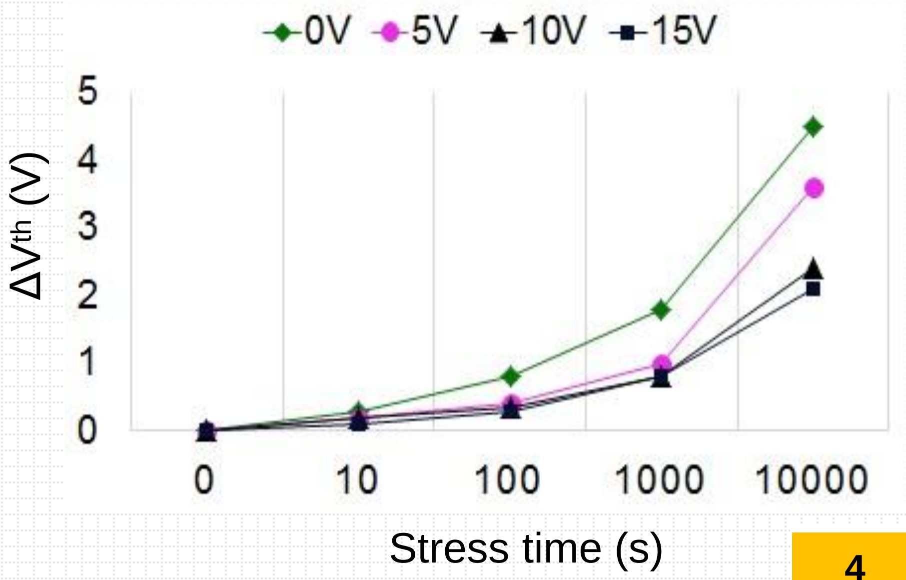
- In deep-submicron, statistical devices induce statistical performances
- Parametric variability induces serious yield loss issues
  - Parametric variability will dominate yield loss
  - Yield affects the total cost of the products



[1] C. Guardiani *et. al*, "An effective DFM strategy requires accurate process and IP pre-characterization", *DAC* 2005

Aging Effects & Parameter Degradation

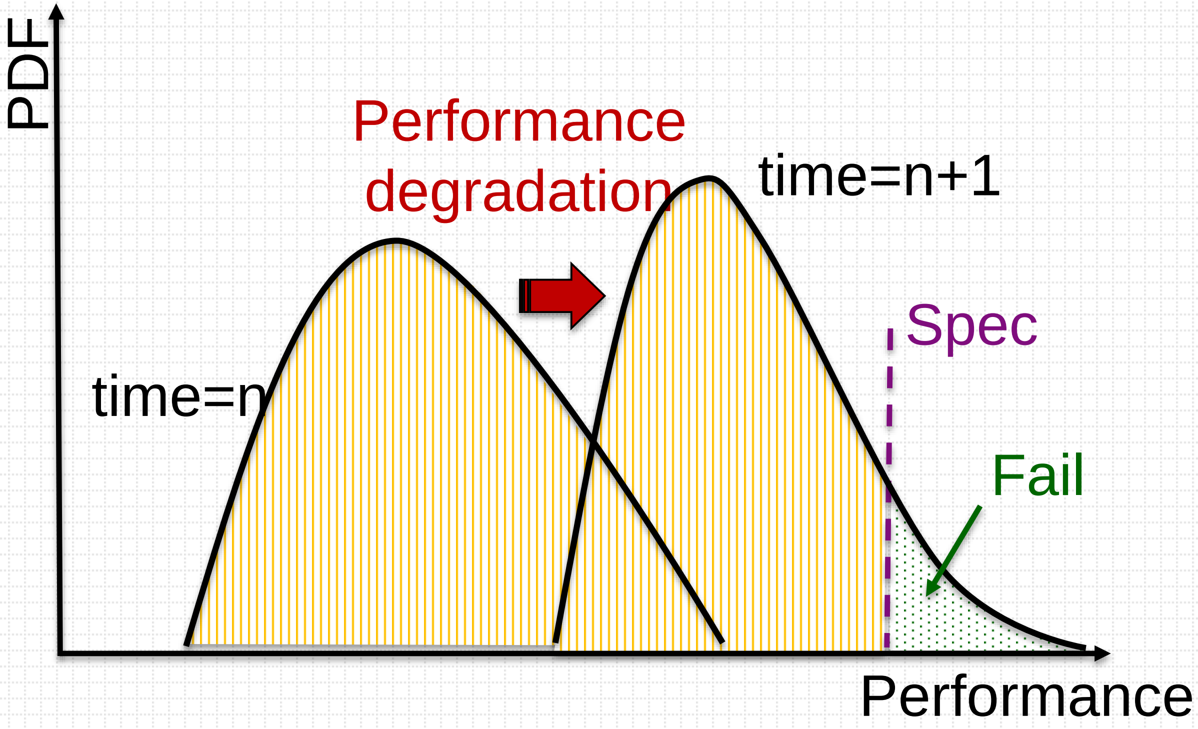
- Aging effects change circuit behavior with time
  - Negative-bias temperature instability (NBTI), hot-carrier injection (HCI)…
- Performance degrades when exposed in the ambient air or under continuous bias-stress
  - V_t is changed a lot over time
  - Impact the circuit performances
  - Reduce yield and reliability significantly
- New technology (ex: flexible TFT) has serious challenge from aging effects
  - V_t is changed a lot in seconds



[2] R. Shringarpure, *et al.*, “Localization of Gate Bias Induced Threshold Voltage Degradation in a-Si:H TFTs,” *IEEE EDL*, vol. 29, no. 1, pp. 93–95, Jan. 2008.

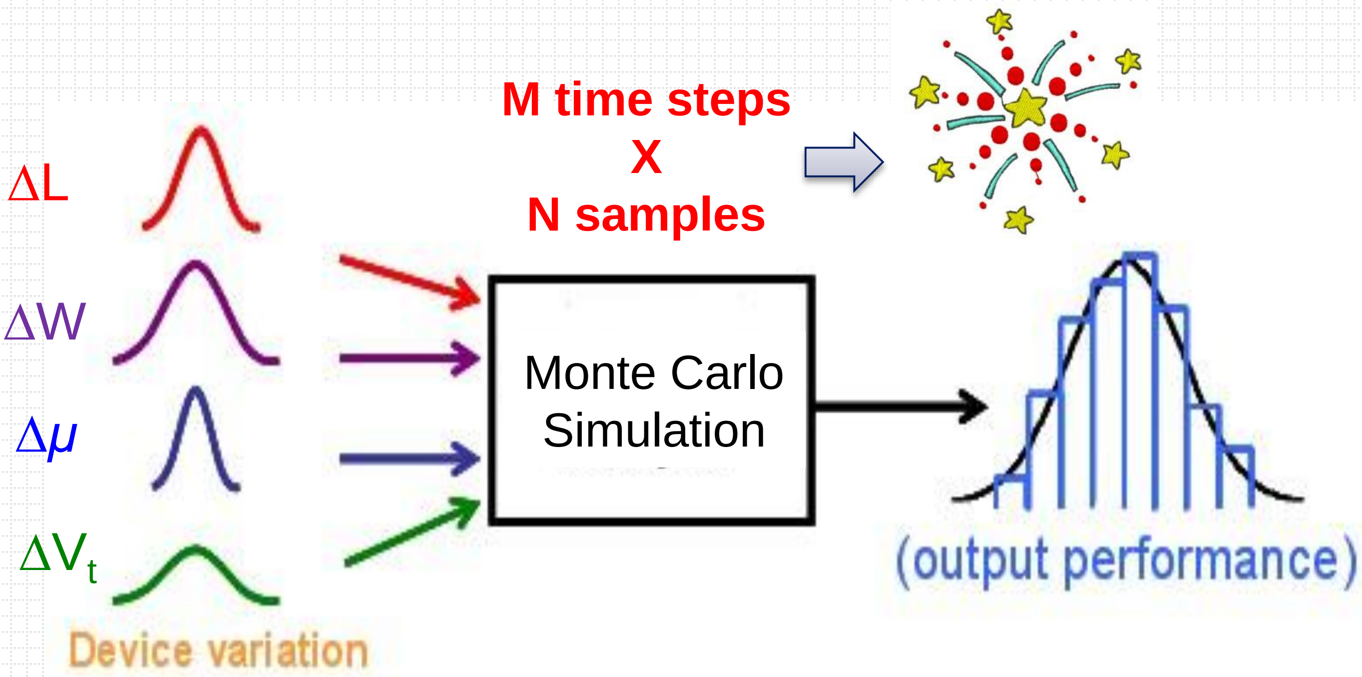
Lifetime Yield

- Lifetime Yield = Process Variations + Aging effects
  - Evaluate the reliability after a period of time
- Lifetime yield analysis often requires iterative circuit performance simulation
  - Evaluating the performance at EACH time step → high cost !!



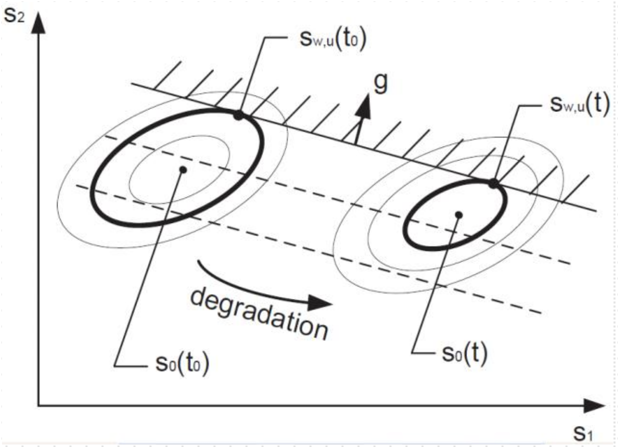
Monte Carlo Simulation

- Still the golden reference for yield analysis
- Simulate a lot of random samples
  - Analyze the performance distribution under process variations
  - High analysis cost
- Infeasible to do the MC analysis at each time step …

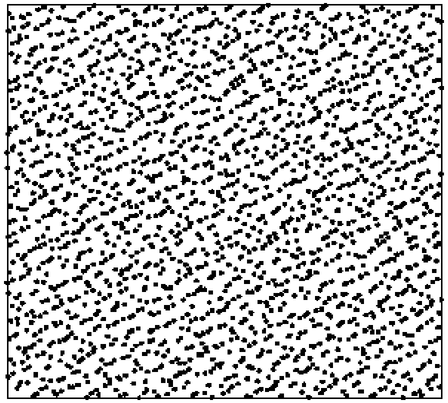


Possible Ways to Reduce Complexity

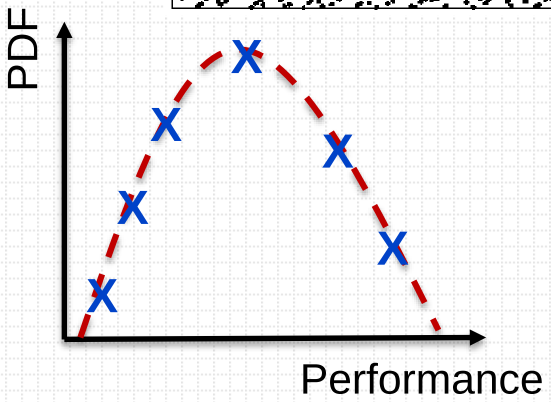
- Simplified simulation model
  - Use behavioral model or equation-based model to predict the circuit performance
  - Simulation time is reduced, but estimation accuracy is also reduced



- Compact sample generation
  - Use special sampling techniques (ex: QMC or LHS) to generate the samples for MC simulation
  - Due to the fast convergence property, the required number of samples can be reduced



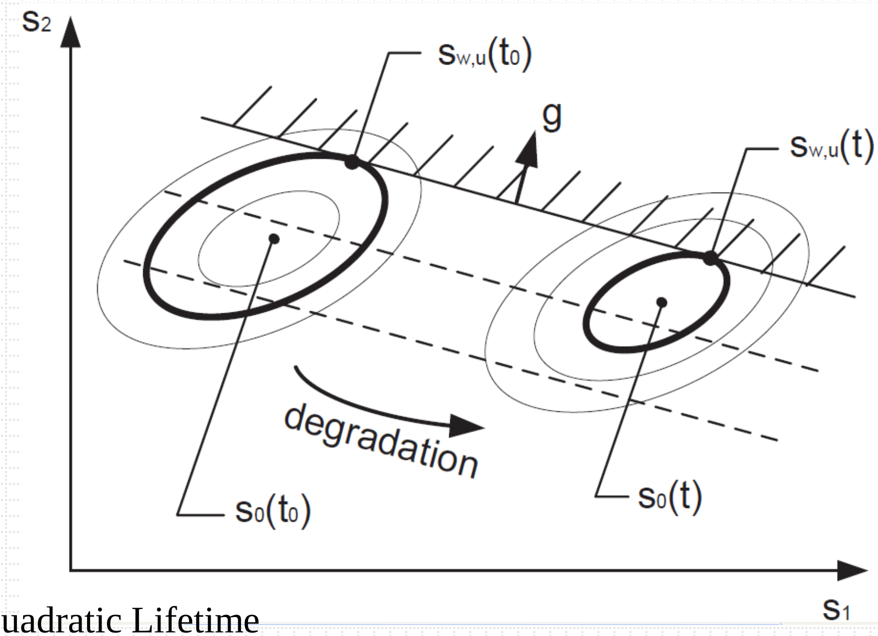
- Performance distribution estimation
  - Also called stochastic modeling technique
  - Use the results of a few samples to estimate the whole probability distribution



Quadratic Model for Lifetime Yield

- Use equation-based model to predict the performance distribution after a given period of time
  - Pretty fast estimation without iterations
  - Non-linear aging effects are hard to be predicted → large error exists

$$y(t) = y(t_0) + \underbrace{\frac{dy(t)}{dt}\Big|_{t0}}_{\text{Sensitivity}} \cdot \underbrace{(t - t_0)}_{\text{Distance}} + \frac{d^2y(t)}{dt^2}\Big|_{t0} \cdot (t - t_0)^2$$

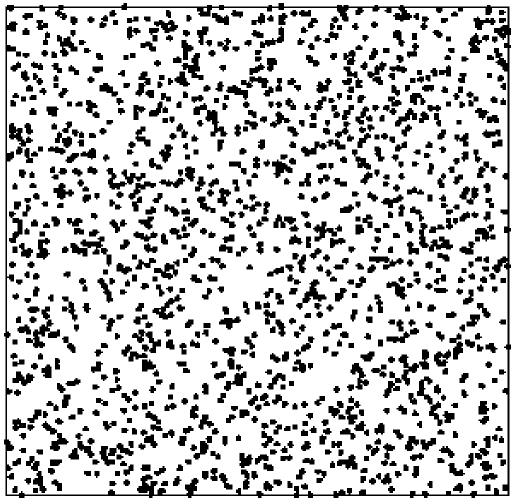


[3] X. Pan, *et al.*, "Reliability Analysis of Analog Circuits Using Quadratic Lifetime Worst-Case Distance Prediction," in *Proceedings CICC*, pp. 1–4, Sep. 2010.

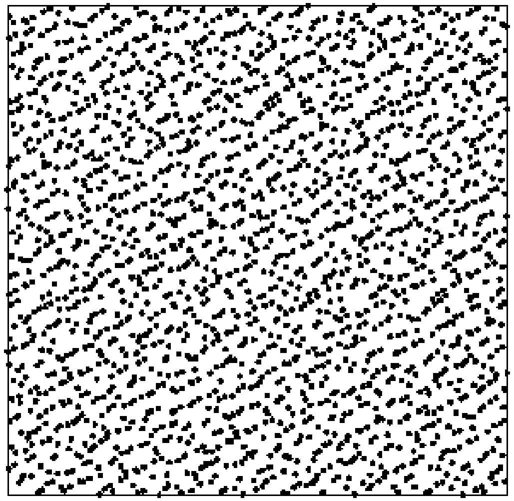
Compact Samples: QMC & LHS

- Quasi Monte Carlo (QMC) method generates low-discrepancy sequences based on specific pseudorandom numbers
- Latin Hypercube Sampling (LHS) is a variant of QMC method
  - Each group in the sampling space contains only one single sample
  - Guarantee all the samples with low dependence
- Control the sample distribution for fast convergence
  - Less samples are required to reach the same accuracy → speedup !!

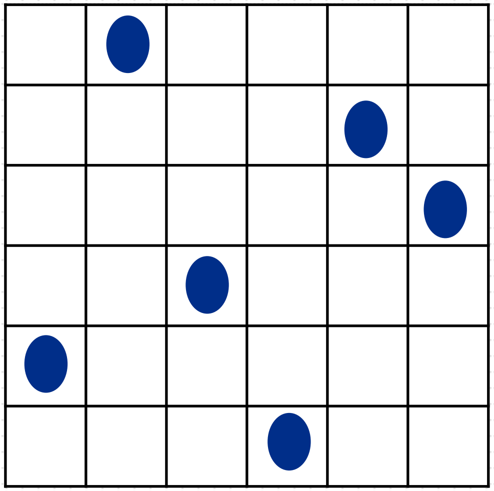
Random



Quasi-random



Latin Hypercube

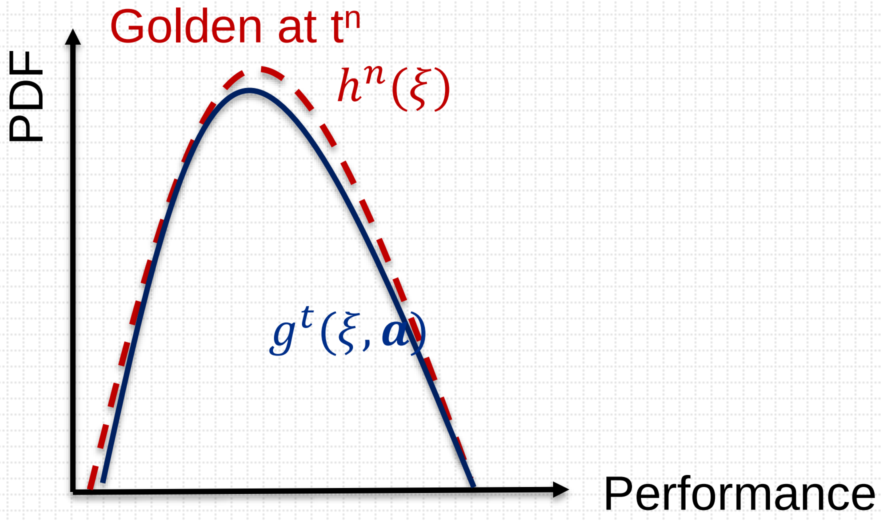


Stochastic Behavioral Modeling

- Moment matching-based method
  - A fast way to estimate the probability distribution with less samples
  - Calculate the probabilistic moments as

$$m_p^k = \frac{1}{N} \cdot \sum x_i{}^{k}$$

  - Solve the resulting nonlinear equation system to obtain residues a_i and poles b_i of *h(t)*, which is the *pdf(x)*



Probabilistic moments of N samples

$$m_p^k = \int_{-\infty}^{\infty} x^k pdf(x) dx$$

match

Time moments of LTI system h(t)

$$m_t^k = \frac{(-1)^k}{k!} \int_{-\infty}^{\infty} x^k h(x) dx$$

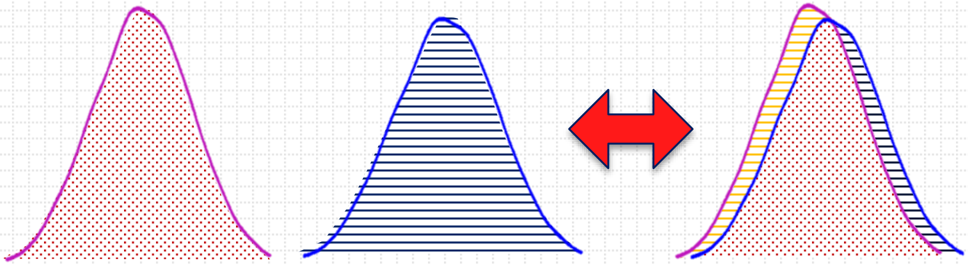
$$\begin{bmatrix} \frac{1}{b_1} & \frac{1}{b_2} & \cdots \\ \frac{1}{b_1^2} & \cdots & \cdots \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \\ \cdot \\ \cdot \\ \cdot \end{bmatrix} = \begin{bmatrix} m_1 \\ m_2 \\ \cdot \\ \cdot \\ \cdot \end{bmatrix}$$

$$pdf(x) = \sum_r a_i \cdot e^{b_i \cdot y_p}$$

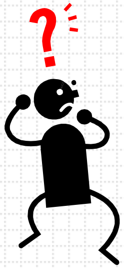
[4] F. Gong *et al.*, “Stochastic Behavioral Modeling and Analysis for Analog/Mixed-Signal Circuits,” TCAD, Jan. 2013.

"Incremental" Sampling for Aging Analysis

- Circuit behavioral is not changed dramatically at each time step during aging analysis
- **Reuse most of samples** and incrementally update a small portion of samples
  - Reduce #simulations for aging analysis significantly



- How to keep the randomness property of samples ?
  → follow the **LHS property** to ensure fast convergence
  - Each row and each column has only one sample !!



- **Stochastic modeling** is adopted to further reduce the samples for estimating the performance distribution
  - Incremental moment matching is proposed in this work

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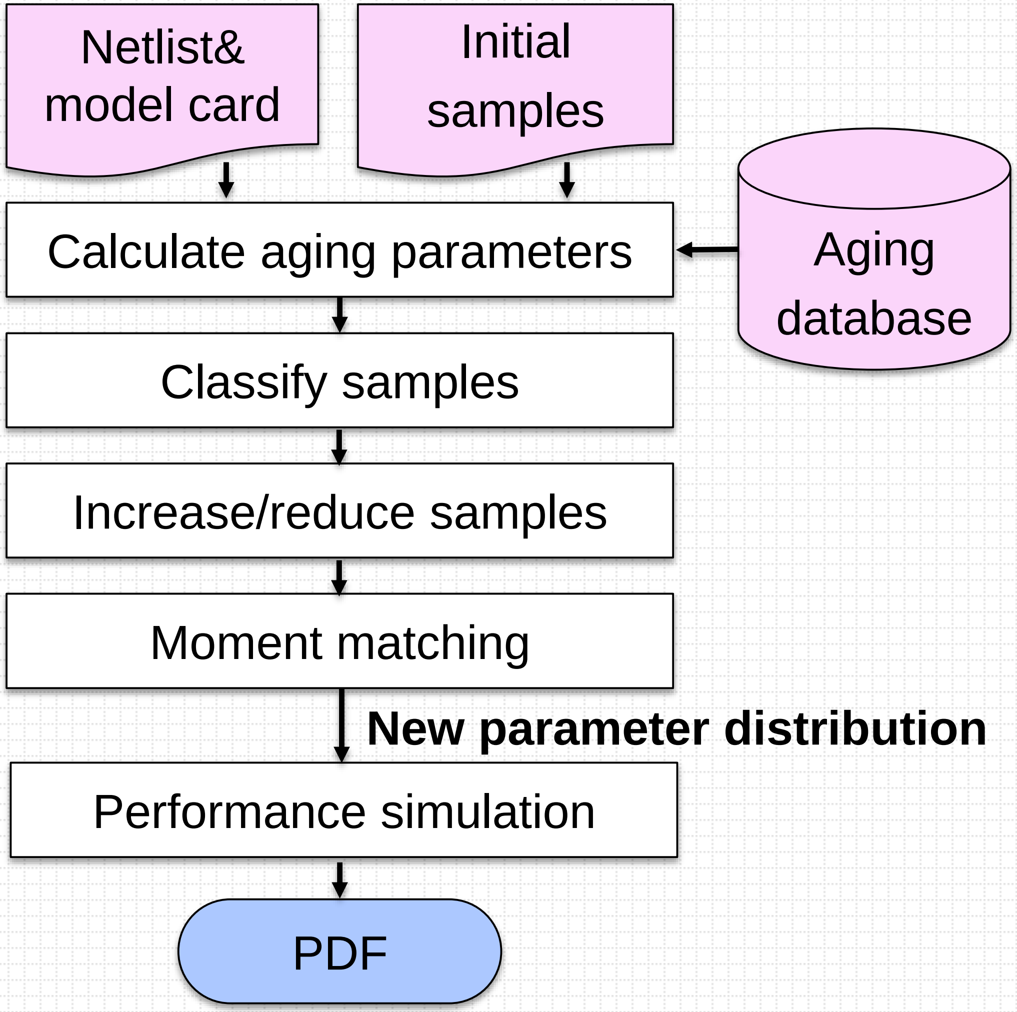
Flowchart of Incremental LHS

- Input
  - Initial samples
  - circuit database
- Output
  - Aging PDF
- Aging Model
  - Exponential Equation [5]


Netlist& model card

Initial samples

Aging database

Calculate aging parameters

Classify samples

Increase/reduce samples

Moment matching

New parameter distribution

Performance simulation

PDF

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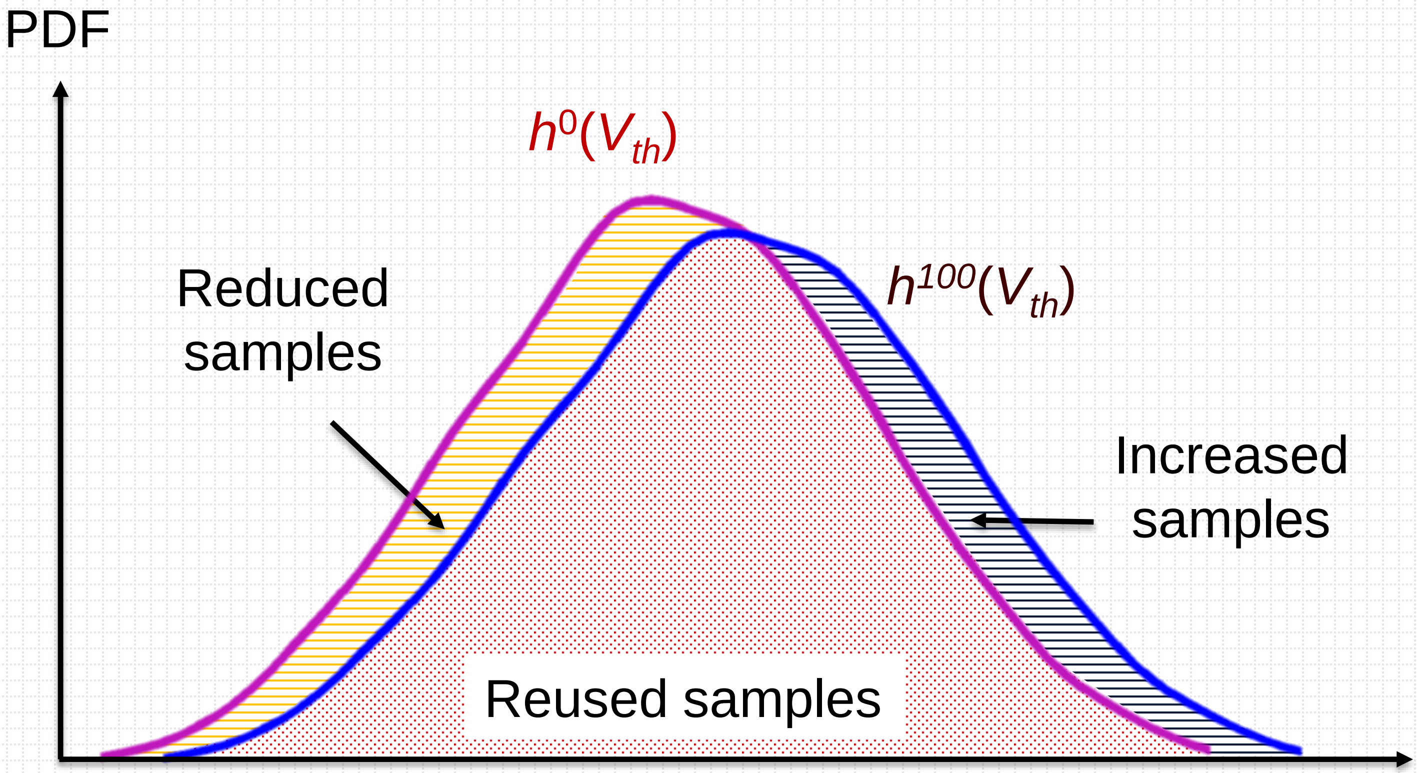
[5] S.-E. Liu, et al., "Estimate threshold voltage shift in a-Si:H TFTs under increasing bias stress," *IEEE TED*, vol. 56, no. 1, pp. 56-59, Dec. 2009.

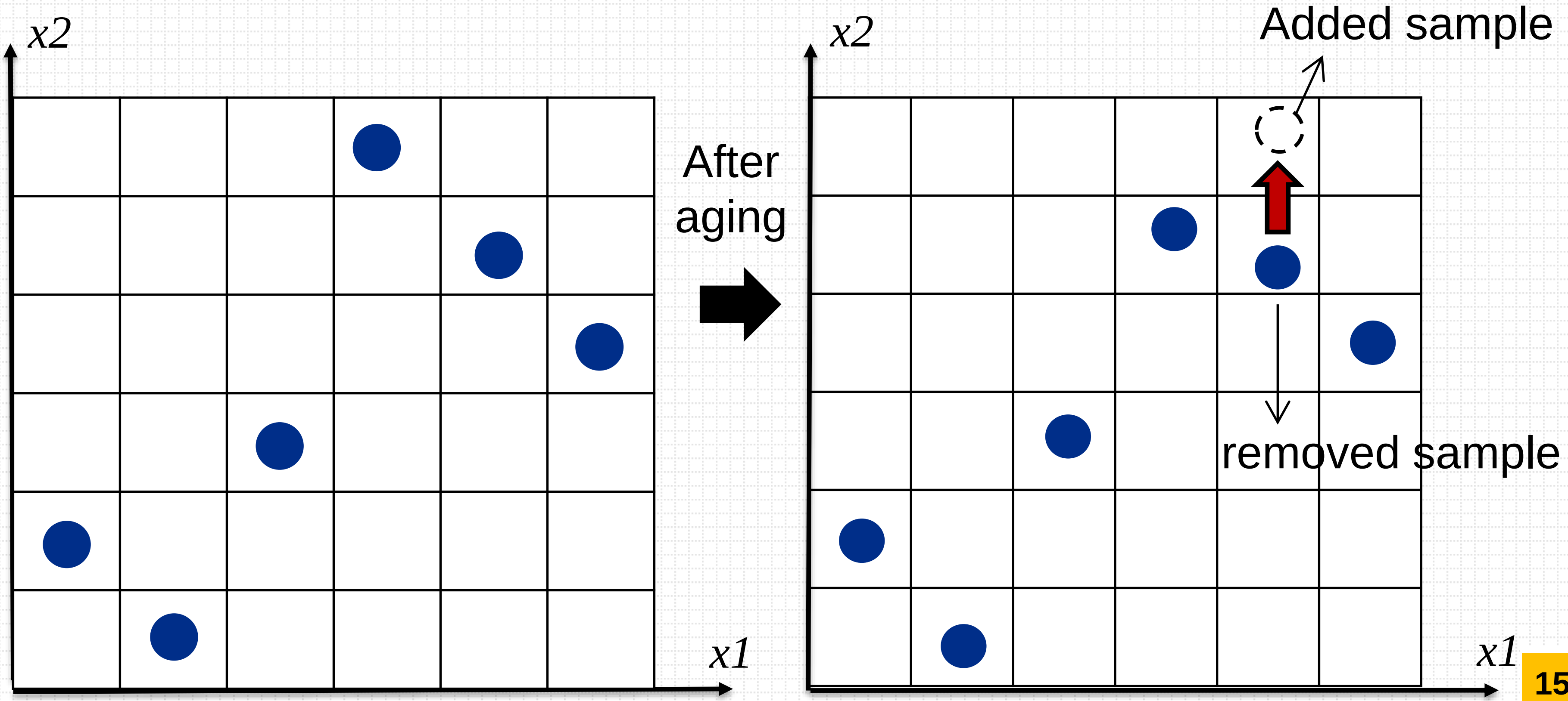
Sample Classification

- The purpose of sample analysis:
  - Reuse the majority of samples
  - Remove some redundant samples
  - Add few new samples



Proposed Incremental LHS Method

- The performance of each sample may be changed after aging
  - Modify performance by estimation to consider aging
- Add/remove samples to keep the LHS property
  - Check each row and each column for required/redundant samples
- Most of the samples are reused !!



Incremental Moment Matching

- Not all the calculations need to be redo
  1) **Re-**Calculate the probabilistic moments as
     - $m_{p\_old}^k = \frac{1}{N} \cdot \sum_{old} x_i^{\ \ k}$
     - $m_{p\_new}^k = \frac{1}{N} \cdot (\sum_{old} x_i^{\ \ k} - \sum_{reduced} x_i^{\ \ k} + \sum_{increased} x_i^{\ \ k})$
     - Only need to consider the “incremental” samples rather than all the samples
  2) **Re-**Match to time moments m_t^k
  3) **Re-**Solve the nonlinear system and obtain the new *pdf(x)*

Probabilistic moments of N samples

$$m_p^k = \int_{-\infty}^{\infty} x^k pdf(x)dx$$

match

Time moments of LTI system h(t)

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$$\begin{bmatrix} \frac{1}{b_1} & \frac{1}{b_2} & \cdots \\ \frac{1}{b_1^2} & \cdots & \cdots \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \\ . \\ . \\ . \end{bmatrix} = \begin{bmatrix} m_1 \\ m_2 \\ . \\ . \\ . \end{bmatrix}$$

$$pdf(x) = \sum_r a_i \cdot e^{b_i \cdot y_p}$$

[4] F. Gong *et al.*, “Stochastic Behavioral Modeling and Analysis for Analog/Mixed-Signal Circuits,” TCAD, Jan. 2013.

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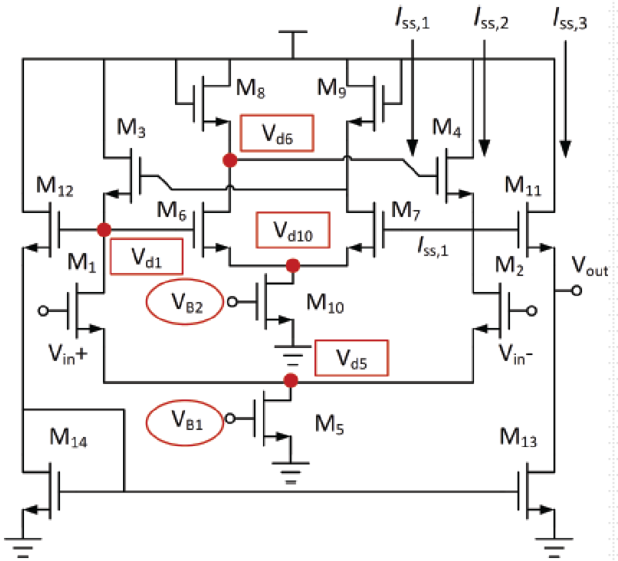
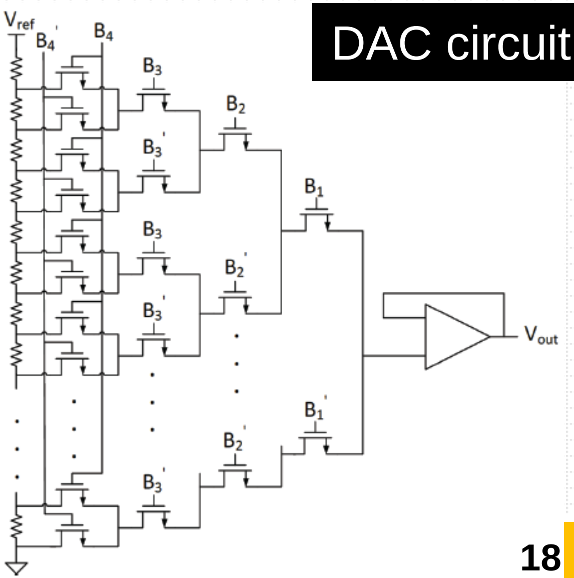
Experimental Environment

- Perform on PC with Intel 2-core 2.50GHz CPU and 2GB memory
- Demonstrated with flexible TFT to observe clear aging effects
  - ITRI a-Si 8μm technology
  - OPA circuit with flexible TFTs [6]
  - 4-bit digital-to-analog convertor (DAC) [6]
- Methods for comparison
  - MC simulation
  - Quadratic model [3]
  - Proposed incremental LHS method

OPA circuit



DAC circuit



[3] X. Pan, *et al.*, “Reliability Analysis of Analog Circuits Using Quadratic Lifetime Worst-Case Distance Prediction,” in *Proceedings CICC*, pp. 1–4, Sep. 2010.
[6] Y. Tarn, *et al.*, “An amorphous-silicon operation amplifier and its application to a 4-bit digital-to-analog converter,” in *IEEE JSSC*, pp. 1028–1035, May. 2010.

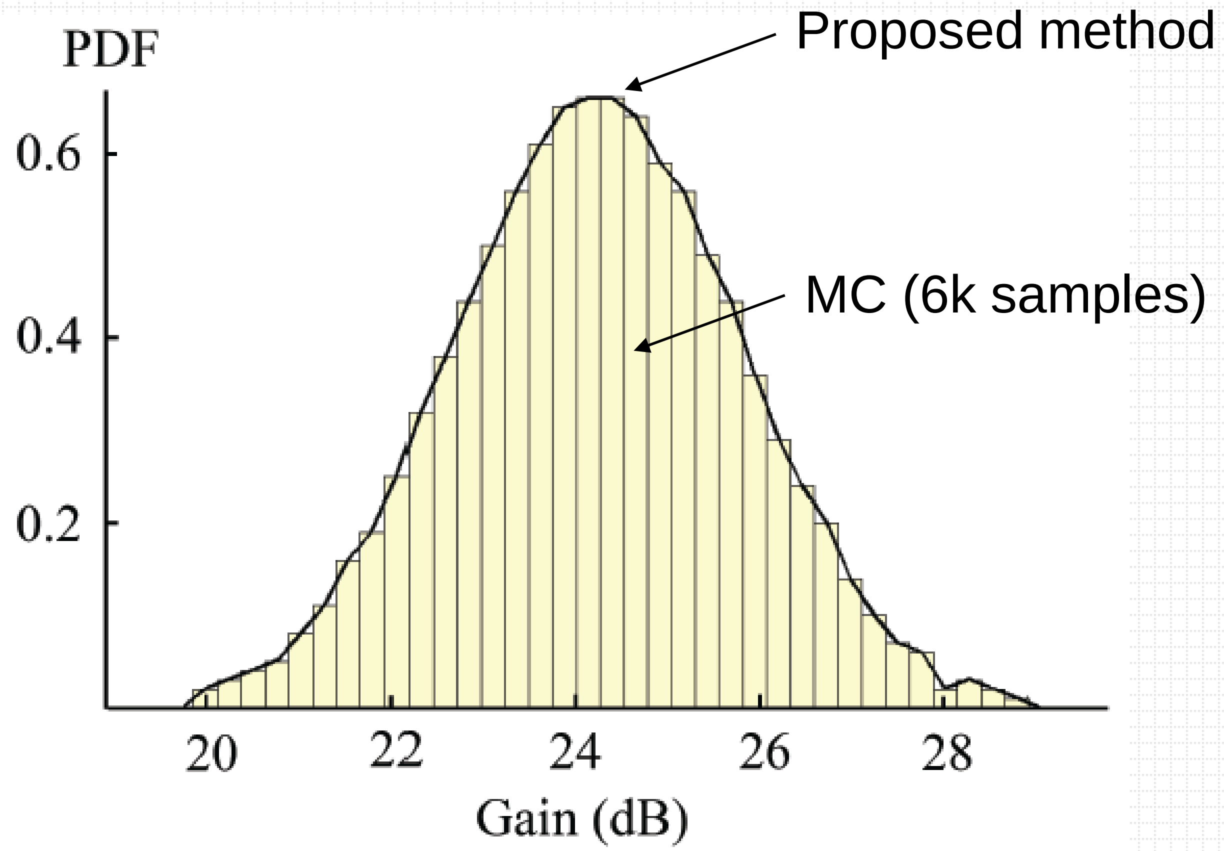
Result Comparison of OPA Circuit

- iLHS achieves 243x average speedup from t=0s to t=10000s
- The accuracy of quadratic model is decreasing over time
  - The prediction error of the non-linear aging rate

Time (s)		MC (6k)	MC (3k)	Quad.	Proposed
0	Accuracy	100%	98%	99%	99%
	# samples	6000	3000	2000+2000	500
100	Accuracy	100%	97%	92%	99%
	# samples	6000	3000	-	13
1000	Accuracy	100%	97%	85%	99%
	# samples	6000	3000	-	16
10000	Accuracy	100%	97%	74%	99%
	# samples	6000	3000	-	29
Overall	Accuracy	100%	97%	83%	99%
	# samples	60000	30000	4000	702
	Speedup	1x	2x	150x	85x

Performance Distribution of OPA Circuit

- Proposed method achieves 99% accuracy with all time step configurations
  - Because of the property of LHS can be kept at all time



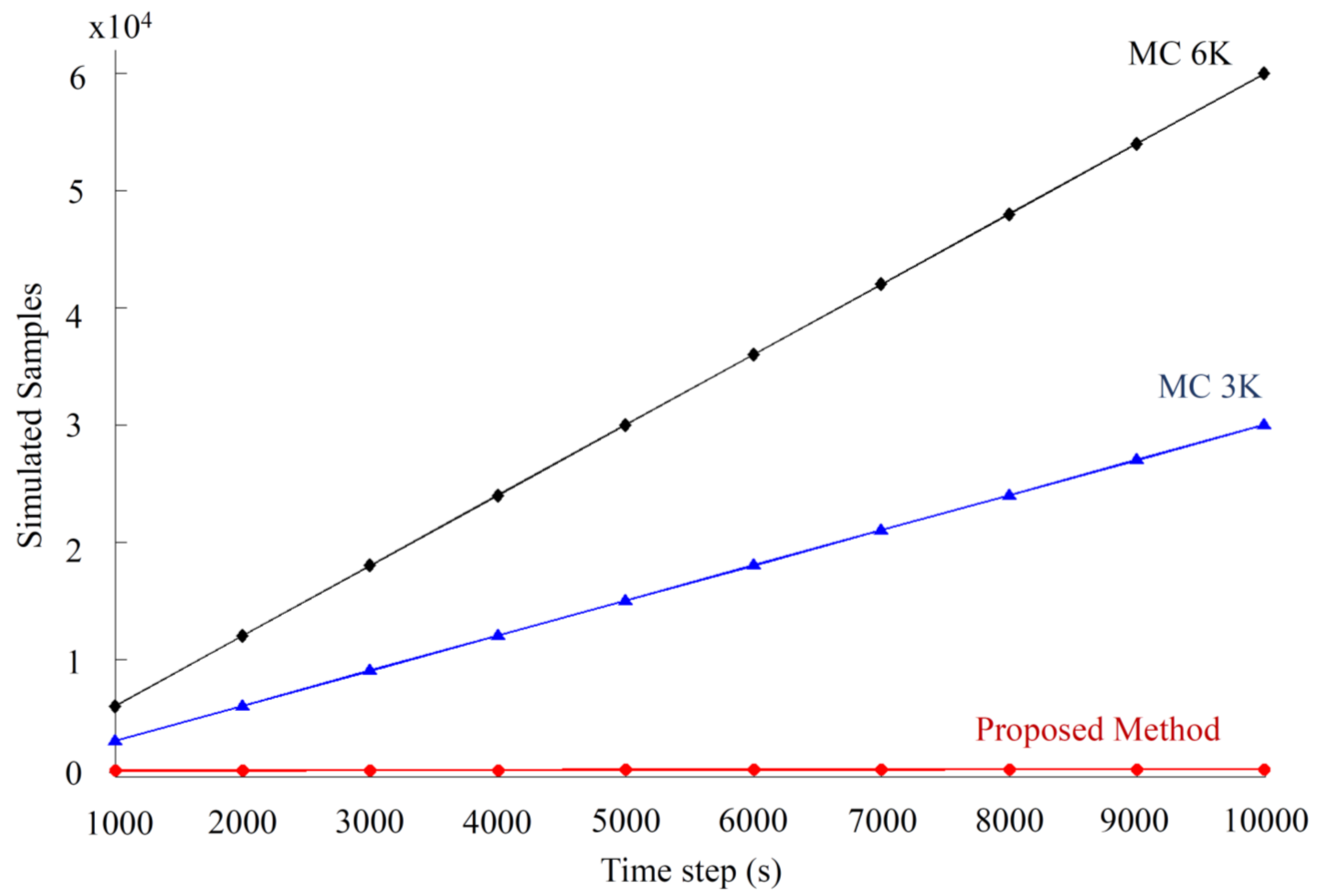
Result Comparison of DAC Circuit

- iLHS achieves 242x average speedup from t=0s to t=10000s
- The accuracy of quadratic model is still low
  - The prediction error of the non-linear aging rate

Time (s)		MC (6k)	MC (3k)	Quad.	Proposed
0	Accuracy	100%	98%	98%	99%
	# samples	6000	3000	2000+2000	500
100	Accuracy	100%	97%	89%	98%
	# samples	6000	3000	-	12
1000	Accuracy	100%	96%	82%	98%
	# samples	6000	3000	-	19
10000	Accuracy	100%	96%	73%	99%
	# samples	6000	3000	-	33
Overall	Accuracy	100%	97%	82%	98%
	# samples	60000	30000	4000	770
	Speedup	1x	2x	150x	78x

Reduction on Simulation Samples

- Only hundreds of samples are required to re-simulate in proposed incremental LHS method
  - **85x** ↑ speed up



Conclusions

- Incremental LHS method is proposed for aging analysis
  - Aging effects change the circuit behavior gradually
- Only a small portion of samples are incrementally updated at each time step in aging analysis
  - Reuse previous samples to greatly reduce the simulation efforts
- Stochastic modeling is adopted to further reduce #samples
  - Incremental moment matching is also proposed in this work
- Experimental results achieve 85x speedup over traditional reliability analysis method with similar accuracy
  - Demonstrated on OPA and DAC circuits

Thanks for your listening !!! ☺

Email: jimmy@ee.ncu.edu.tw