Fast and Accurate Stochastic Analysis for Custom Circuits

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Track 4: Power and Reliability Analysis and Optimization

1. BACKGROUND AND MOTIVATION

As the process technology scales down to nanometer, analog and mix-signal (AMS) circuits are more prone to process, voltage and temperature (PVT) variations. Stochastic circuit analysis simulates circuits while considering PVT variations. It helps circuit designer to shift the post-silicon verification to pre-silicon phase debug, which is more cost friendly and also significantly shortens the time to market. Two important problems related to stochastic circuit analysis are studied.

First, it is important to to understand the stochastic behavior of circuit performance during the pre-silicon phase. Second, analyzing the rare failure probability for highly duplicated circuits, such as memory cell, I/O cell, are crucial to the reliability. To solve these two problems, the "gold standard", Monte Carlo (MC) method, needs to run expensive transistor-level simulation to get the accurate probabilistic distribution (e.g., PDF and CDF) of circuit performance. A lot of efforts have been spent to reduce the runtime of a single simulation [1, 2, 3].. However, MC is still extremely inefficient because millions of samples need to be simulated to capture one single failure when the failure is a rare event. Therefore, fast non-MC approach is highly desired.

Three approaches are proposed in this abstract to solve the aforementioned two problems:

1. A maximum entropy (MAXENT) based approach is proposed to model the "arbitrary" behavioral distributions [4].
2. To perform the high sigma analysis, we proposed a high dimensional importance sampling (HDIS), which maintains stable even when there are a large number of process variation parameters [5].
3. A Rare-Event microscope (REscope) has been developed to handle the high sigma analysis while the failure samples are distributed in multiple failure regions. REscope also adopted an algorithm to reduce a high dimensional problem to a lower one [6].

2. BEHAVIOR MODELING VIA MAXIMIZING ENTROPY (MAXENT)

To approximate the circuit behavior considering process variation, asymptotic probability extraction (APEX) and Point Estimation based Method (PEM) approximate the PDF of circuit performance via padé approximation by matching the probabilistic moments to time moments. However, both of them are numerically instable when high order moments is used. A stabler algorithm to analysis of the stochastic circuit behavior is highly desired.

Alternatively, we proposed a maximum entropy (MAXENT) based approach to model the "arbitrary" behavioral distributions for AMS circuits. The exact behavioral distribution can be approximated by a product of exponential functions with different Lagrangian multipliers. The closest approximation can be obtained by maximizing entropy subject to moment constraints, leading to a nonlinear system. Classic Newton's method is used to solve the nonlinear system for the Lagrangian multipliers, which can further recover the arbitrary behavioral distribution of AMS circuits. Extensive experiments on many circuits have validated that the proposed method is more robust and offers up to 35% higher accuracy compared to PEM, and up to 195x speedup compared to MC method.

3. HIGH DIMENSIONAL IMPORTANCE SAMPLING (HDIS)

In high sigma analysis, the existing importance sampling approaches can be successfully applied to low-dimensional problems with small number of variables but, in general, perform poorly in high dimension. Under high dimension, the joint PDF of the shifted distribution, say $g(X)$, could be very small and it makes the likelihood ratio $f(X)/g(X)$ ($f(X)$ is the original distribution) suffer from numerical errors, which leads to a inaccurate failure rate estimation.

High Dimensional Importance Sampling (HDIS) is proposed solved this problem. It avoids directly using the likelihood ratio $f(X)/g(X)$ to estimate the failure probability, but using a different approach. In details, HDIS first constructs a new subset of the sampling space that dominates the failure region for memory circuits and can be efficiently estimated with a few samples. Then, the failure probability of memory circuits can be evaluated by the product rule of conditional probability within this sampling subset space.

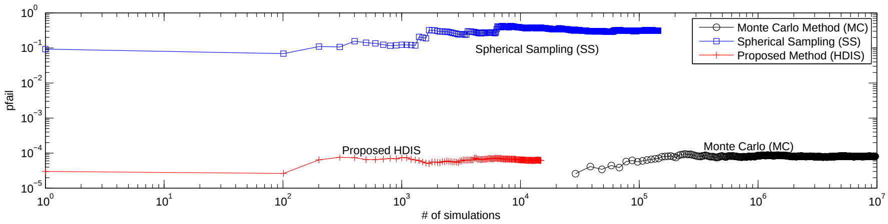


Figure 1: Evolution comparison of the failure probability estimation using different methods.

More importantly, the estimation from the proposed method is proved to be always bounded in high dimensions.

On a 117-dimension circuit, the classification based method fails to improve the performance by blocking “unlikely to fail” samples, and Spherical Sampling method completely fails to provide reasonable accuracy. Contrastingly, the proposed approach yields accurate result with 364X speedup over Monte Carlo. The result is illustrated in Figure 1.

4. RARE-EVENT MICROSCOPE

The HDIS solves the problem of high dimensionality. However, non of these approaches considers the possibility that fail samples fall in multiple failure regions. Therefore, an approach that could accurately analysis the rare failure probability with high-dimensional process variations and multiple separate failure region is highly desired.

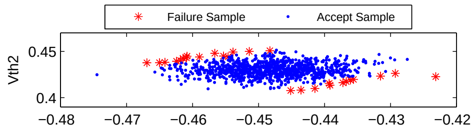
To solve this problem, we proposed the Rare-Event Microscope (REscope), which consists of three major components, (1) parameter pruning, (2)classification, and (3) tail distribution estimation. It prunes the less useful process variation parameters in a high-dimensional problem by considering the contribution of each parameter to the performance metrics. Furthermore, we applied a nonlinear SVM classifier which is capable to identify multiple separate failure regions. Due to the nonlinear classifier, we can zoom in and only look at the failure regions to model the circuit performance distribution of those likely-to-fail samples into a generalized pareto distribution (GPD), which is known as a good model of the tail of the PDF.

REscope is evaluated on a charge pump circuit in the PLL design. In an illustrative example with only 2 process variation parameters, we demonstrate that the REscope could be able to identify multiple failure regions, while HDIS and Statistical Blockade fails. The comparison result is presented in Figure 2.

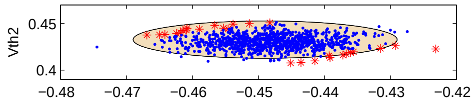
On the same circuit, but with 108-dimension process variation parameters, the proposed method outperforms the importance sampling approach and is 389x faster than the Monte Carlo approach. Moreover, it estimates the failure rate accurately, while the importance sampling totally fails because the failure region is not correctly captured.

5. SUMMARY

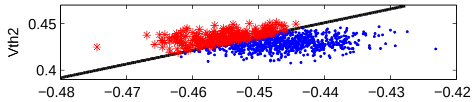
In this abstract, I presented three pieces of research I have done on statistical circuit modeling and analysis. For the stochastic behavior analysis, the MAXENT models the stochastic behavior of a circuit more efficiently and accurately than existing moment matching based works. For high sigma analysis, the HDIS solves the numerical stability problem of conventional importance sampling approach and exhibits a high efficiency. At last, REscope is proposed to handle the high sigma analysis while the failure samples are distributed in multiple failure regions. REscope is also able to reduce a high dimensional problem to a lower on by adopting machine learning based algorithms. [1]



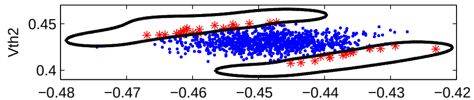
(a) Two separate failure regions



(b) Importance sampling region in HDIS



(c) Classification result in SB



(d) Classification result in REscope

Figure 2: How multiple failure regions are handled in HDIS [5], SB, and REscope [6]

6. REFERENCES

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[6] W. Wu, W. Xu, R. Krishnan, Y.-L. Chen, and L. He, “REscope: High-dimensional statistical circuit simulation towards full failure region coverage,” in *Proceedings of the The 51st Annual Design Automation Conference on Design Automation Conference*. ACM, 2014, pp. 1–6.

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[1] Reference to existing work is not included in this abstract. Only those most related publications of mine are referred.