PHD FORUM

Fast and Accurate Stochastic Analysis for Custom Circuits

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Abstract

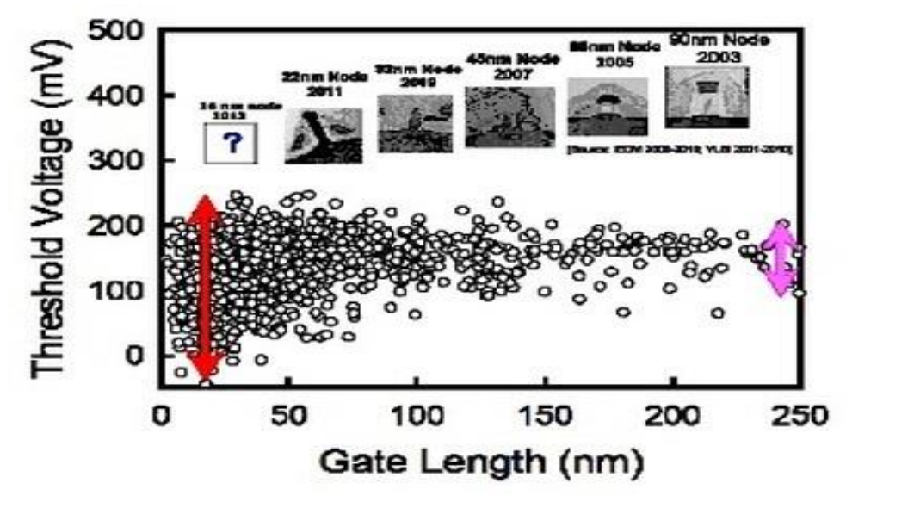
AMS circuits designed with advanced technology are more prone to process, voltage and temperature (PVT) variations.

Stochastic analysis simulates circuits while considering PVT variations. It helps designer to shift the post-silicon verification to pre-silicon phase debug, which is more cost friendly and also significantly shortens the time to market.

In this poster, three approaches are presented to solve the stochastic analysis problem introduced by PVT variation.

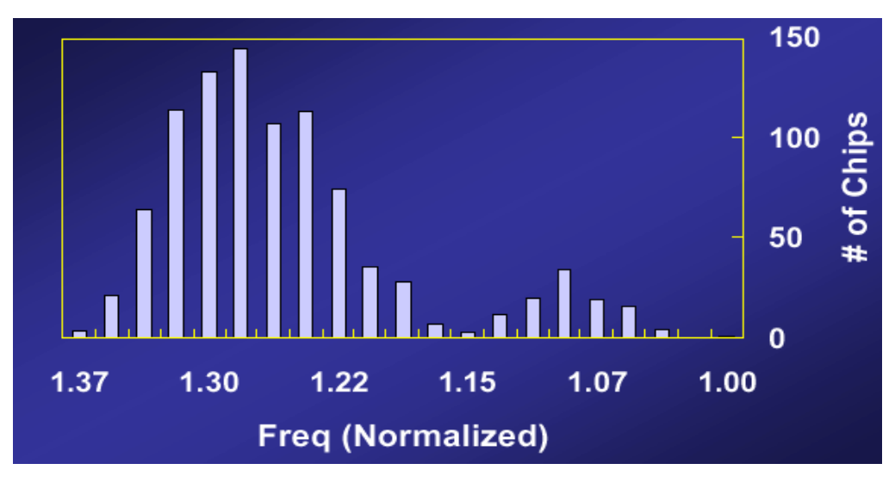
- **MaxEnt**: analyze the stochastic behavior model by maximizing entropy [ISQED'13]
- **HDIS**: high dimensional importance sampling for rare-event analysis [ASPDAC'14]
- **REscope**: high dimensional statistical circuit simulator toward full failure region coverage [DAC'14]

Introduction

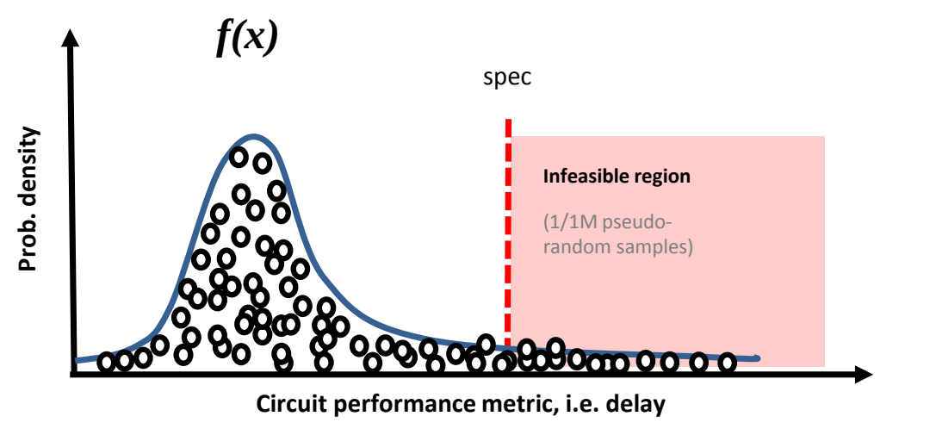


Courtesy of Prof. Yiming Li, National Chiao Tung University, Taiwan

- **PVT Variations (PVT)**
  - –Process Variation, Supply Voltage, Operating Temperature, aging, etc.
  - –Shrinking device size ➔ more prone to PVT variations
- **Why?**
  - –Stochastic Analysis helps designer to debug circuits in the pre-silicon phase, and enhances the yield rate.
- **How** to run it accurate yet efficient?
  - –**Stochastic behavior modeling** of a circuit with light simulation load
  - –**High sigma analysis**: Estimating the rare failure event for yield enhancement



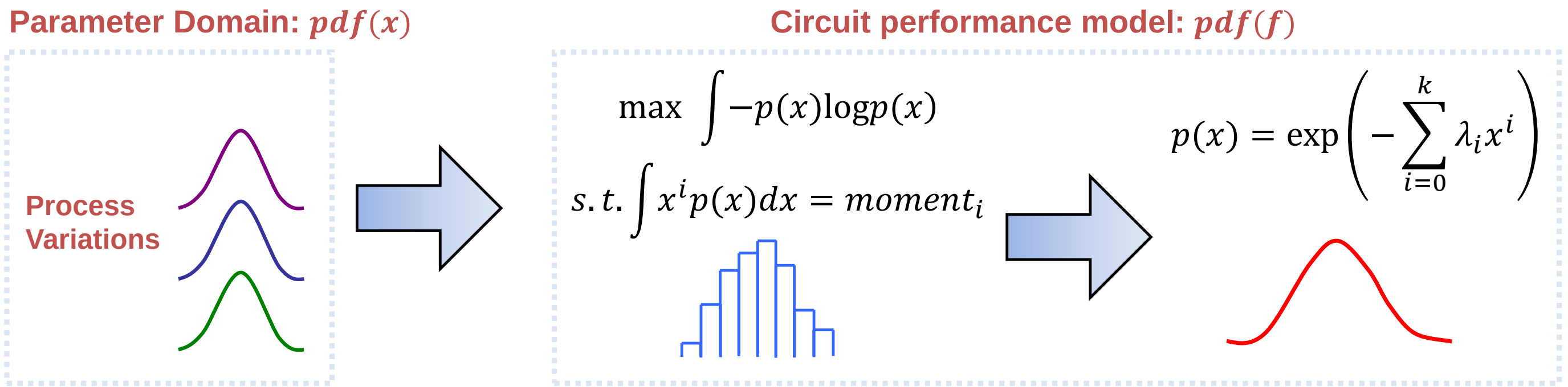
Distribution of operating frequency under process variation [courtesy of UC Berkeley]



Sigma	Probability	Simulation runs¹	Time²
1	0.15866	700	7 secs
2	0.02275	4,400	44.0 secs
3	0.00135	74,100	12.4 mins
4	3.17E-05	3,157,500	8.8 hours
5	2.87E-07	348,855,600	40.4 days

Stochastic Behavior Modeling by Maximizing Entropy (MaxEnt)

- **Statistical Modeling of Performance Distribution**
  - –It is desired to extract the **arbitrary distribution of the circuit performance**, according to the device variations
- **Existing approach**
  - –Monte Carlo (MC):
    - –accurate, but very inefficient
  - –APEX, PEM :
    - –numerical instable
- **MaxEnt Highlights:**
  - –Models the PDF as product of exponential terms
  - –Use optimization approach to find out the parameter in each exponential term
  - –Fast and numerically stable

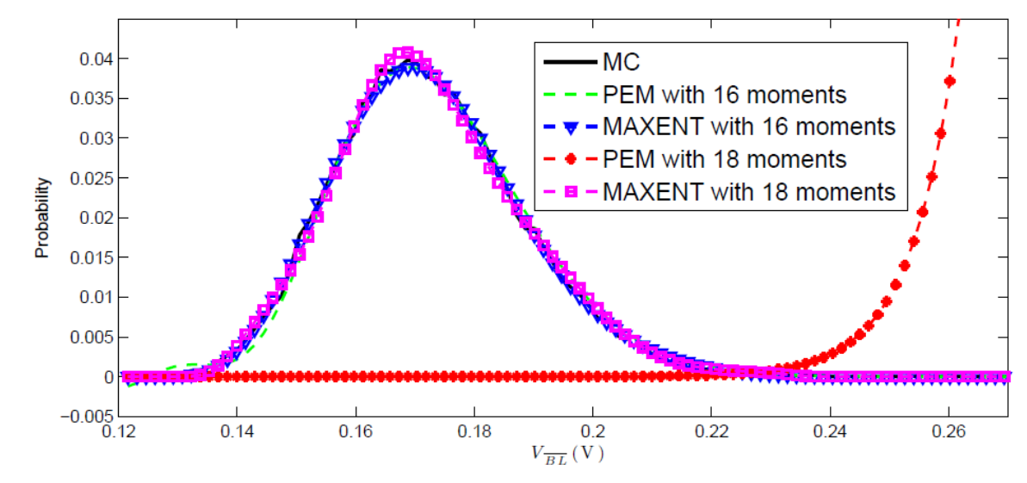


$$\max \int -p(x)\log p(x)$$

$$s.t. \int x^i p(x)dx = moment_i$$

$$p(x) = \exp\left(-\sum_{i=0}^{k} \lambda_i x^i\right)$$

- **Experiment Results on a SRAM-cell circuit**
  - – MaxEnt remains stable on high order moments

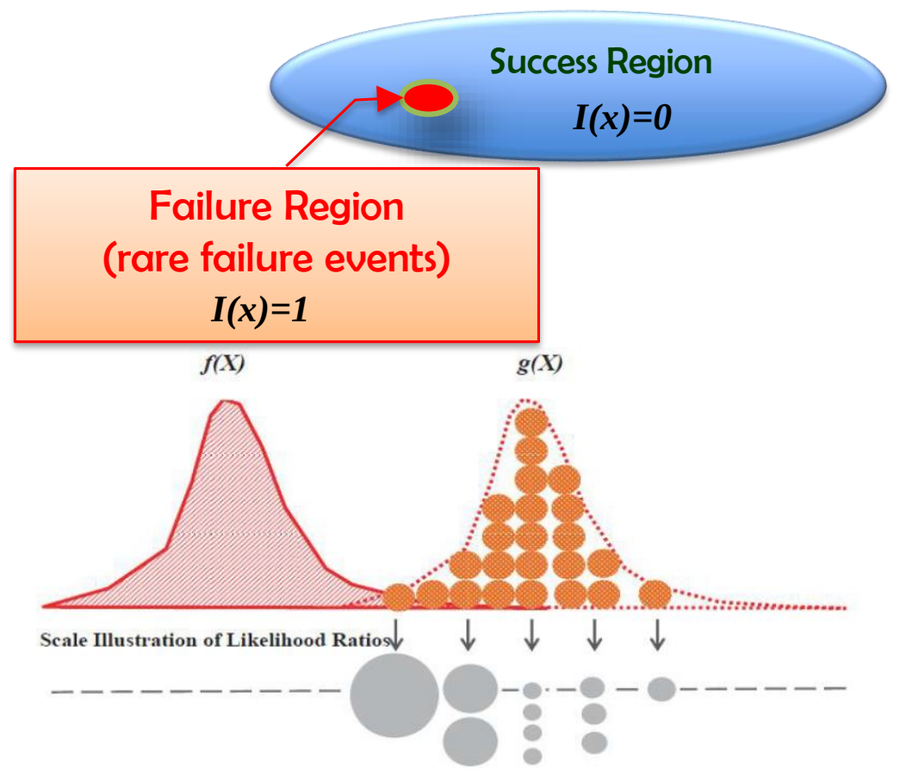


  - – Speedup:

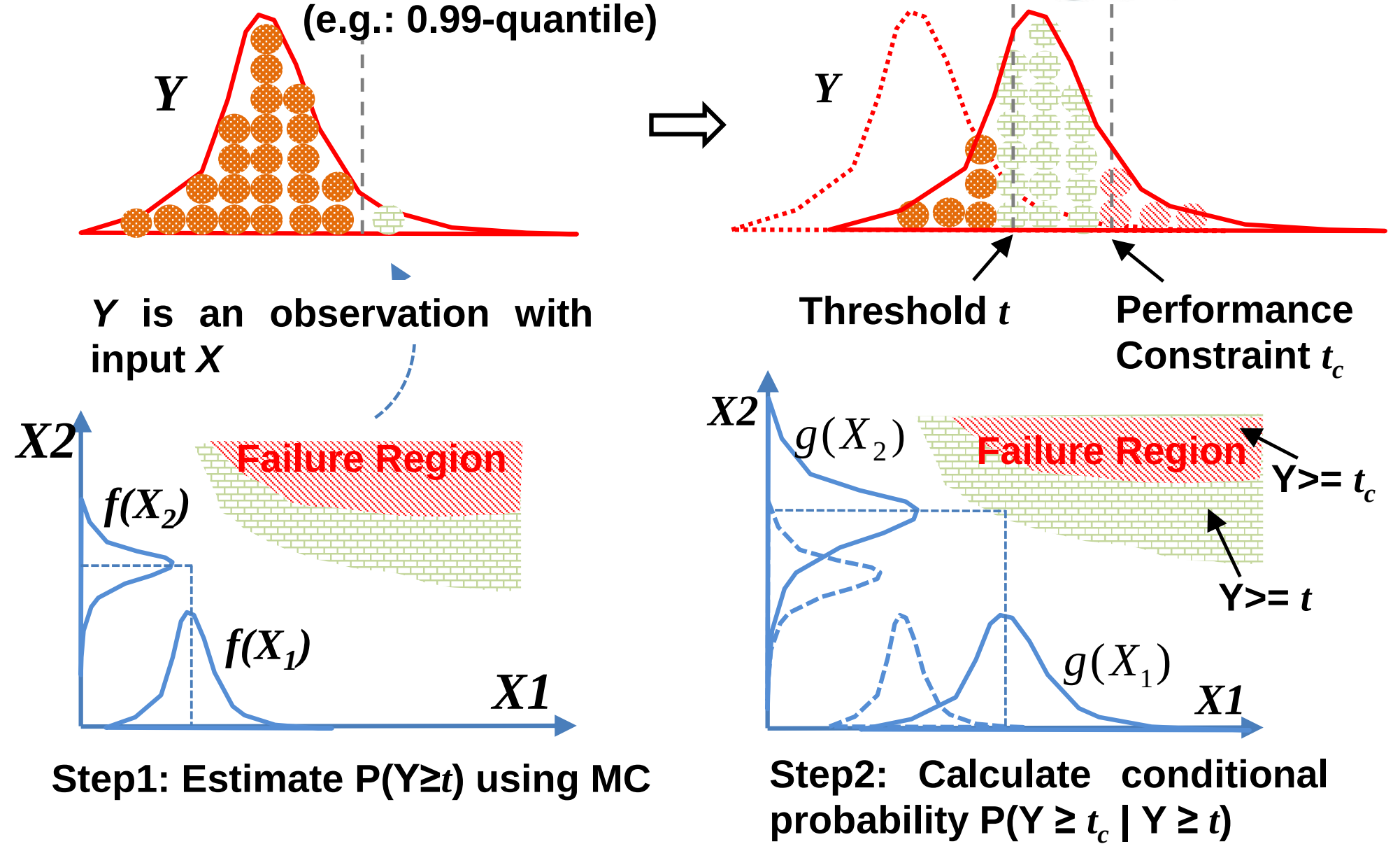
Method	# of simulations	Speedup	Error (%)
Monte Carlo	39,000 (0.47hours)	1x	0%
MaxEnt	200 (11 secs)	195x	3.09%

High Dimensional Importance Sampling (HDIS)

- **Importance sampling and its numerical issue**
  - – Shift the sample distribution to more "important" region
  - – Reweight and estimate the failure probability
    - ➢$prob(Failure) = \int I(x) \cdot f(x)dx = \int I(x) \cdot \frac{f(x)}{g(x)} \cdot g(x)dx$
  - – Weight is unbounded and unstable at high dimension.



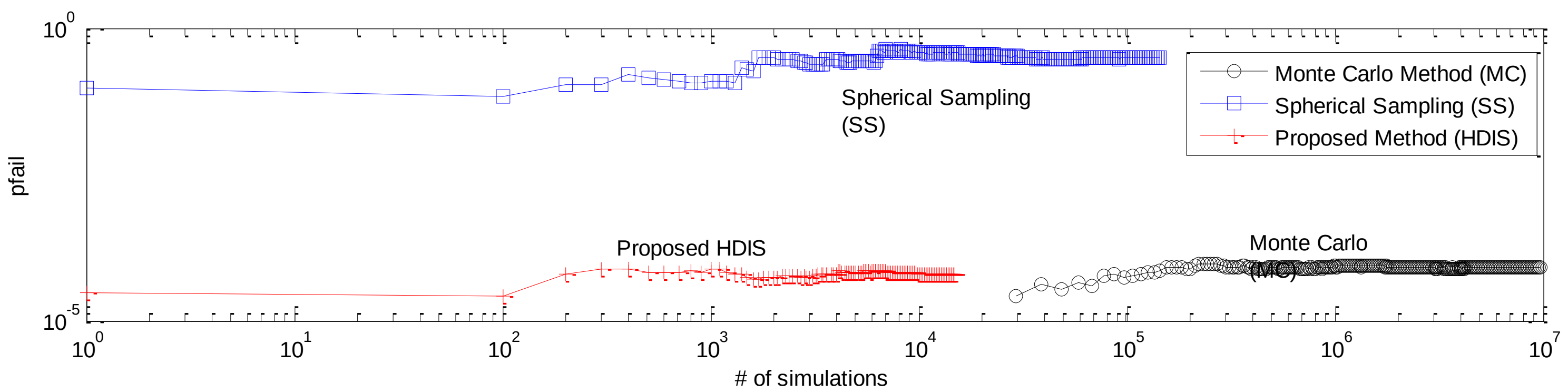
- **Proposed HDIS**
  - – Step1: Build a non-rare region $\{Y|Y \geq t\}$ and evaluate $P(Y \geq t)$ with MC
  - – Step2: Shift the mean to the centroid of the tail (approximated by limited sampling), and estimate the conditional probability $P(Y \geq t_c|Y \geq t)$
  - –Failure probability:
    - ➢$prob(Failure) == P(Y \geq t) * P(Y \geq t_c|Y \geq t)$



Step1: Estimate P(Y≥t) using MC

Step2: Calculate conditional probability P(Y ≥ t_c | Y ≥ t)

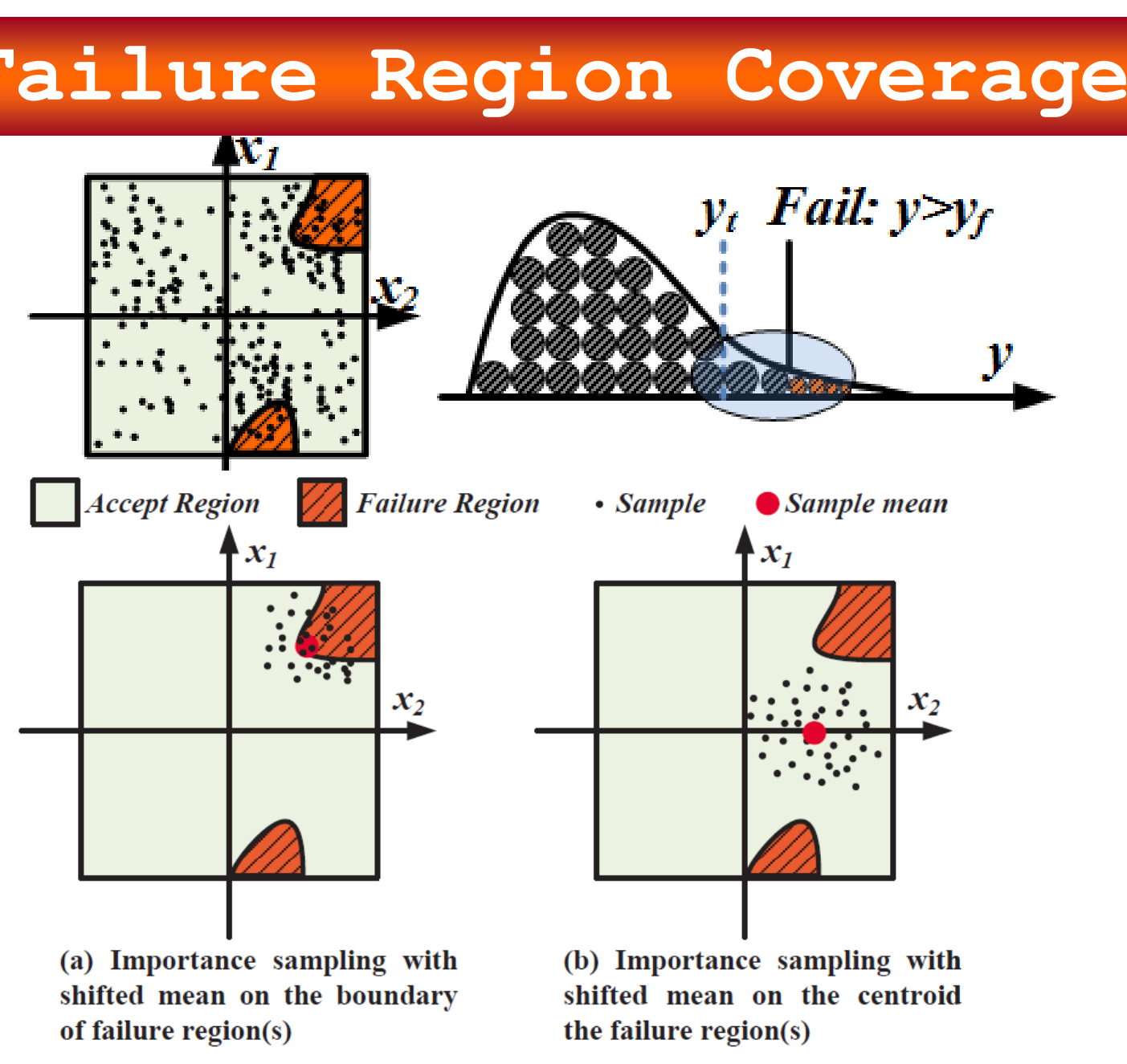
- **Experiment Results** (on a differential amplifier with 117 variation parameters)
  - –Estimated failure rate and efficiency



Target failure probability		Monte Carlo (MC)	Spherical Sampling (SS)	Proposed Method (HDIS)
8e-3	prob:(failure)	8.136e-4	0.2603	7.861e-3 (3.4%)
	#sim. runs	4.800e+4 (24X)	16000 (8X)	2000
8e-4	prob:(failure)	8.044e-4	0.2541	8.787e-4 (9.2%)
	#sim. runs	4.750e+5 (36X)	8.330e+4 (6.4X)	1.300e+4
8e-5	prob:(failure)	8.089e-5	0.3103	8.186e-5 (1.2%)
	#sim. runs	5.156e+6 (346X)	1.430e+5 (10X)	1.500e+4

Rare-Event Microscope for Full Failure Region Coverage

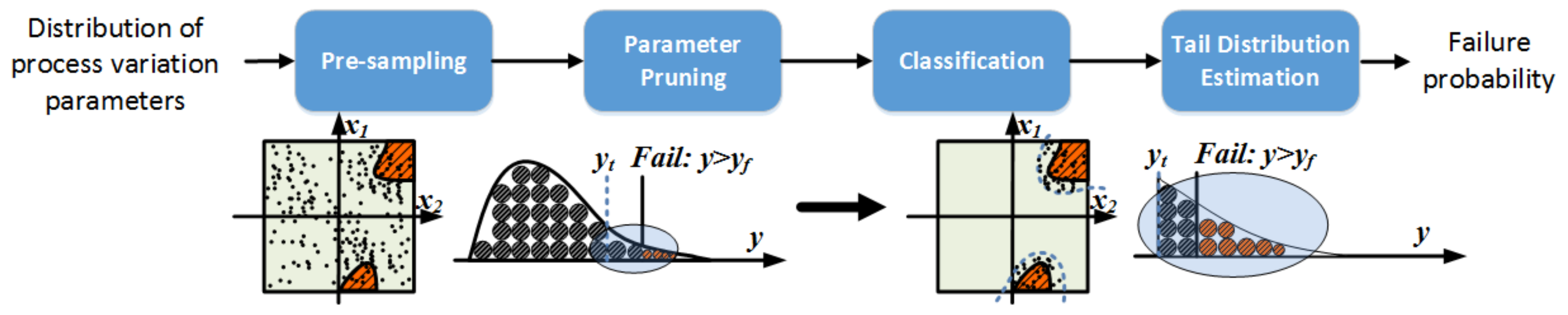
- **What if failed samples fall in multiple disjoint regions?**
  - – Mean shifting based approaches will behave like this
  - – Train a **classifier** Block the "unlikely-to-fail" samples might be helpful, but linear classifier (such as Stochastic Blockage) cannot make it



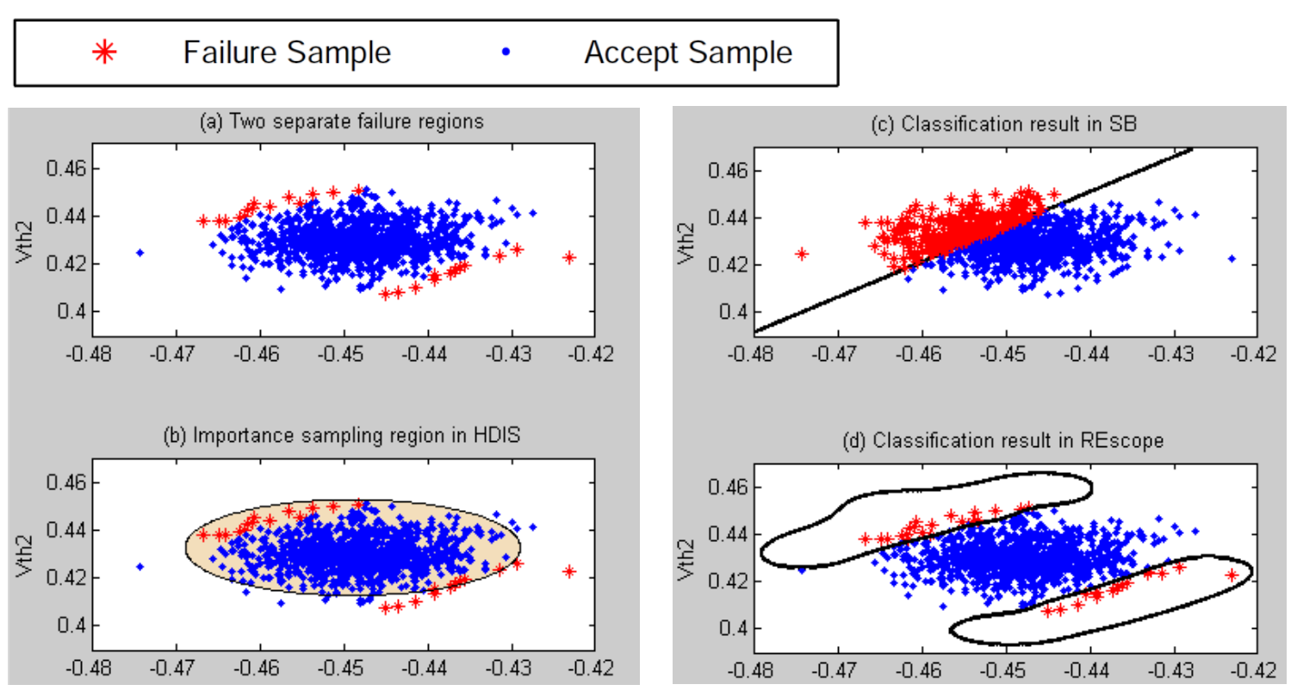
(a) Importance sampling with shifted mean on the boundary of failure region(s)

(b) Importance sampling with shifted mean on the centroid the failure region(s)

- **Proposed Rare-Event Microscope (REscope)**
  - – Identify **multiple failure regions**
  - – Handle **high dimensional** problems
  - – Approximate the tail as a **generalized pareto distribution (GPD)**

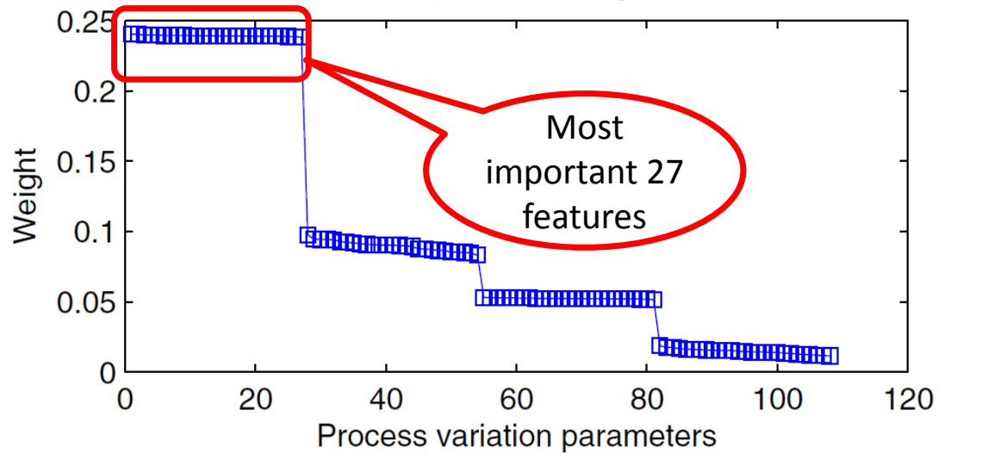


- **Experiment Results**
  - – Charge pump circuit in PLL
  - – A illustrative setup with 2 variation parameters

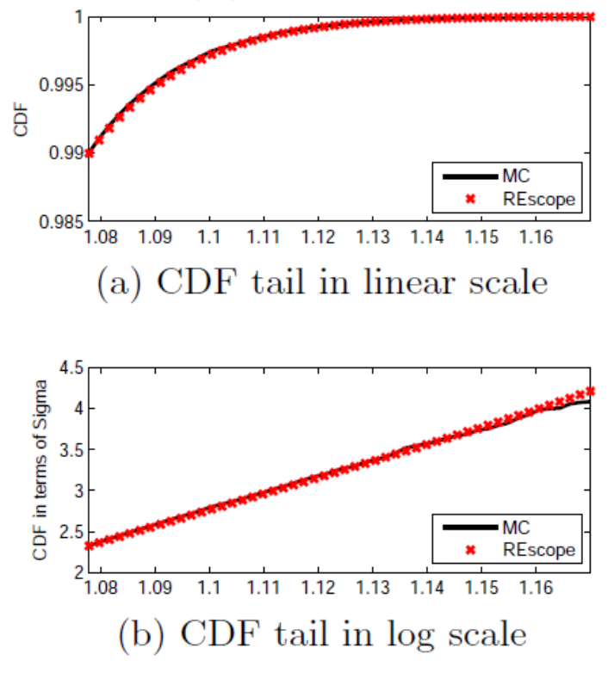


  - – A high dimensional setup (108 variables)

Parameter pruning:



CDF approximation:



(a) CDF tail in linear scale

(b) CDF tail in log scale

Table 1: Comparison of the accuracy and efficiency on charge pump circuit

	Monte Carlo (MC)	Importance sampling (HDIS)[12]	Proposed approach (REscope)
failure probability	2.279e-5 (0%)	1.136e-3	2.256e-5 (+1.05%)
#sim. runs	1.4e+6 (389x)	2e+4 (5.6x)	3.6e+3 (1x)

References

Related publications:

- [DAC'14] Wei Wu, Wenyao Xu, Rahul Krishnan, Yen-Lung Chen, Lei He. "REscope: High-dimensional Statistical Circuit Simulation towards Full Failure Region Coverage", in 51st ACM/IEEE Design Automation Conference, 2014.
- [DAC-WIP'14] Yen-Lung Chen, Wei Wu, Lei He, and Chien-Nan Liu "Stochastic Behavioral Modeling of Analog Circuits with Reliability and Variability for the Applications on Flexible Electronics", 51st ACM/IEEE Design Automation Conference Work-In-Progress workshop, 2014.
- [ASPDAC'14] Wei Wu, Fang Gong, Gengsheng Chen, and Lei He, "A fast and provably bounded failure analysis of memory circuits in high dimensions," in 19th Asia and South Pacific Design Automation Conference, 2014.
- [ISQED'13] Rahul Krishnan, Wei Wu, Fang Gong, and Lei He, "Stochastic behavioral modeling of analog/mixed-signal circuits by maximizing entropy", ISQED'2013.

Collaborators: Fang Gong (Cadence); Wenyao Xu (SUNY Buffalo) Yen-Lung Chen, Chien-Nan Liu (National Central University)